TRANSACTIONS OF
THE
OPTICAL SOCIETY

VOL. XXIV. 1922-23 No. 3.

ON THE PROPERTIES OF PITCH USED IN WORKING OPTICAL GLASS

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(Issued from the Research Laboratory of Messrs Taylor, Taylor and Hobson, Ltd.)

MS. received, 6th January, 1923. *Read and discussed, 11th January and 8th February*, 1923.

ABSTRACT. The paper deals with pitch as a mountant for lenses during the operations of grinding and polishing. It is shown that the viscous nature of pitch entails a number of rather intricate consequences in workshop practice, and that these phenomena require the viscosity to lie between certain well-defined limits. It must be sufficiently high to prevent measurable displacement under the external forces applied in polishing and sufficiently low to relieve rapidly the severer internal strains produced by thermal expansion and contraction. It is also shown that there is considerable difficulty in conserving the necessary properties of pitch even at relatively low temperatures, and means for meeting these difficulties are indicated.

The paper is divided into four sections: Section I outlines previous knowledge of the subject; Section II deals with the physical properties of pitch and the alterations produced by low-temperature heating, in so far as these affect its workshop utility. Section III describes a number of workshop phenomena and correlates them with the physical properties described in Section II. In Section IV a brief account is given of some new methods for the mechanical handling of pitch, based on the foregoing studies.

INTRODUCTION.

THE properties of optical glass in so far as these concern the ultimate performance of the glass in the completed optical system have been made the subject of meticulous study for many years: but the properties of the glass in so far as they concern the workshop processes of manufacture and tooling have met with relatively little attention. As a consequence the ultimate performance of the glass can be predicted with great accuracy by previous calculation, and little room is left for empiricism,

while the behaviour of the glass in the workshop remains a mystery understood by none, and only controlled in some measure by the craftsmanship of the skilled workman. If this is the case with the glass itself, it is natural that other workshop materials of less value and forming no part of the finished product should have received still less attention, and, so far as I am aware, little or no information is available on the properties of pitch, whether used for polishers or for a backing to support the lens and to attach it to a holder or "runner."

Practically speaking all optical glass (excluding spectacle glass, which in the strict sense does not rank as optical glass) is polished on polishers of pitch or of wax having properties similar to those of pitch, and most commonly the glass elements—including spectacle lenses—are attached to holders by pitch or some similar cementing material during the fine grinding and polishing operations: and I think it will be generally agreed among workmen that the pitch used for these purposes is the cause of quite as much trouble as is the glass itself.

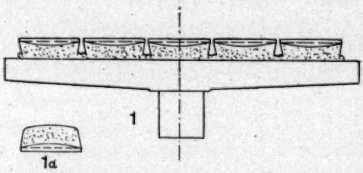

Fig. 1. Block of plano-convex lenses mounted on "runner" by the "pellet" method.

Fig. 1*a*. Lens mounted on pitch by pitch-stick method.

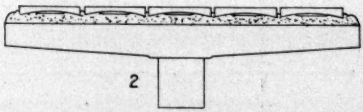

Fig. 2. Block of plano-concave lenses mounted by the "layer" method.

The researches outlined below were carried out during the war period with a view to obtaining a better knowledge of the properties of pitch and to eliminating some of the troubles: in the present communication the properties of polishers will not be considered in any detail, but the subject will be treated with particular reference to pitch as a mounting for a lens during grinding and polishing.

In the simplest case a pitch mounting may be attached to the lens for the purpose of holding it in the hand when polishing a single surface; but more commonly a number of lenses are mounted together on a single holder to form a "block." The term "block" is generally understood in this sense in the optical industry, though in plate glass working it commonly refers to the polisher and not to the work. There are several ways in common use of making up a block with pitch mountings for the lenses; but for practical purposes they fall into two categories, hereinafter called the "pellet" and "layer" methods. The pellet or button method consists in mounting together on one holder a number of lenses prepared with individual backings of pitch as if for single surface working (Fig. 1). The layer method con-

sists in covering the runner as a whole with a uniform covering of pitch and forcing the lenses into it by pressure and heat (Fig. 2).

The second method is in general not entirely satisfactory: it is quicker under some circumstances and acts moderately well for spectacle lenses, where great accuracy of thickness is not essential. In mounting ordinary convex lenses of small edge thickness by this method, however, it commonly happens that the pitch is squeezed out in the interstices and gives trouble by coming into contact with the grinding tool or polisher. This may result in coarse abrasive being retained by the pitch and being set free to cut the lenses later, or it may result in the skinning of the polisher surface, an effect particularly troublesome with felt polishers. Again, if concave lenses are mounted by this method the lenses cannot readily be separated from the pitch after polishing by chilling or refrigeration. The thermal expansion of pitch is greater than that of glass, and on freezing, the pitch contracts tightly round the rim of the glass, and if the latter be weak it may be broken off. In the case of the pellet-mounted lens, the pitch has only to contract to part readily from the glass, and this is generally by far the cleanest and most satisfactory way of separating the easily damaged surface.

The pitch used for polishers is most frequently a wood pitch, while coal tar pitch is most often used for backings. This is by no means always the case, and there appear to be no other reasons for the choice than that coal tar pitch is not readily obtainable sufficiently soft for polishers, while wood pitch as marketed requires much baking to make it hard enough for mountings: it is also relatively expensive.

Section I. The Origin of Pitch and Recognised Tests for Pitch.

The Origin of Pitch.

When complex organic materials such as wood or coal are heated above their decomposition point (destructive distillation) they break up into a great number of substances whose nature varies with the conditions of distillation.

Wood tar ("Stockholm" or "Archangel" tar) is prepared chiefly in the great pine forests of Northern Europe, in Scandinavia, Finland, and Russia, from the stools and roots of the Scotch fir and allied trees. The destructive distillation of these woods produces gas, tar, pyroligneous acid, acetone, wood spirit, and charcoal—all of them products of commercial value. The charcoal, of course, remains in the still; the distillate separates into three layers, the lowest being the tar and the creosote. Above that is a layer of water containing acetone, wood spirit, etc., and the lighter hydrocarbons float on the top. The tar, being drawn off, may be distilled again; at a temperature of about 110° C. water and acetic acid are driven off and from 120°–230° C. the heavier or creosote oils; the residue in the still is wood pitch.

Coal tar is a gas-works by-product formed in the destructive distillation of coal. It consists of a great variety of hydrocarbons from benzene (C_6H_6) to the quite indistinguishable complex of the residual pitch. In the redistillation of the tar light oils of an average density ·91 to ·94 come off as the temperature is slowly

raised to 170° C.; about this time the density of the distillate begins to approach 1·0, and the oils which come off from 170°–230° C. are known as the middle or carbolic oils; they contain the bulk of the carbolic acid and naphthalene. From 230°–270° C. we get the heavy or creosote oils, with a density 1·04, after which the anthracene or "green" oils come off till the end of the distillation. The point to which distillation is carried depends on the kind of pitch it is proposed to leave in the still; the longer the distillation is prosecuted, the harder the residual pitch, but operations cease in any case in time to prevent "coking" of the pitch.

Recognised Tests for Pitch.

For the purposes of specifying pitch a number of tests of a rather empirical nature are used by the gas-works chemist and road engineer. The *Twisting Point* test consists in determining the lowest temperature at which a stick of the material can be twisted indefinitely corkscrew-wise without breaking. The *Penetration* test consists in determining the rate at which a solid body, usually a "needle," is forced into the material by a specified load. Experience of a number of penetration tests made with flat-ended needles and with cones of various angles led us to abandon these in favour of the Brinell ball test.

The quantity measured by the Brinell test when applied to pitch is not the same as the "hardness" obtained with the same test on metals. With metals the indentation made by the ball under a given load increases rapidly to a limiting value, until the area of the impression is sufficient to carry the load without further flow: the quantity determined is really the "pressure of fluidity." In the testing of pitch, unless it has been absolutely coked, no such limiting pressure is reached, but flow continues indefinitely. It is necessary therefore, in making an arbitrary "hardness" test of pitch, to specify not only the load, but the time during which it is allowed to act. Subject to this condition comparative "hardness" figures may be obtained for pitch by calculating the hardness as load in kilogrammes per spherical area of the depression in square millimetres. A more detailed discussion of the hardness test is given later.

The chemist's analysis usually states the percentage of *volatile matter*, of *free carbon*, and of *ash*. Volatile matter is the weight of the material that can be driven off by a red heat in a non-oxidising atmosphere. Free carbon is the material that cannot be thus driven off but can be burnt to CO_2 and water in an oxidising flame. Ash is the residual incombustible mineral matter. The quantities are apt to vary a little with the conditions of experiment.

Another chemical test of some importance is the determination of *soluble matter*; the solvents employed may be carbon disulphide, pyridine, solvent naphtha, benzene, etc., and the test is best conducted by the Soxhlet extraction method. The pitch may be "extracted" with all the solvents in turn, and the matter then remaining insoluble is sometimes called the free carbon. Strangely enough the terms "free carbon" and "fixed carbon" mean the same thing.

The workshop tests hitherto in use in the Optical Industry have usually been hardness tests of some sort. A good idea of the hardness of a pitch for polishers

may be, and usually is, obtained by chewing it, but backing pitch is too hard for this purpose, and some more elaborate penetration or hardness test is substituted. Such a test is, however, inadequate, as pitches of very different qualities may have equal hardnesses. For example, a pitch that has become too hard by prolonged baking may be re-softened by the addition of anthracene oil. But this mixture does not restore the pitch to the same constitution as it had previously at the same degree of hardness, and there is in general a marked difference in the qualities of the re-softened pitch and the pitch prepared by "straight distillation."

The pitch is usually purchased softer than required for use. This is necessary inasmuch as gas-works do not set themselves out specially to prepare pitch to an accurate specification for glass-working purposes and successive consignments are likely to vary considerably. The pitch is brought, by heating, to a standard hardness which is measured in some shops and in others left to the judgment of the workmen or of some other responsible person. It may then be mixed with solid matter, such as wood ash or ochre, or with resin or similar materials, and by further baking the mixture is finally brought to a new standard hardness. Thereafter it is placed for use in the shops, being usually contained in open vessels heated by a gas ring, or, more rarely, by a steam jacket. Layer mountings are made by ladling out a sufficient amount of pitch. Pellets are usually prepared by the pitch-stick method, or occasionally by casting the pellets in moulds.

The pitch-stick method is an artistic process, which consists in thrusting a stick of about 1 inch diameter into the molten pitch and withdrawing a "bloom" of it on the end. This is chilled somewhat by rolling on a cold metal plate till it is plastic without being molten, and at the same time it is drawn into a truncated conical form. The end of the chilled cone is re-liquefied over a flame and the lens attached to it: the pitch is then worked out into intimate contact with the lens over its whole rear surface, is cut through behind the lens with a pair of scissors, and the quantity thus left attached to the glass is then moulded by hand to the right shape and allowed to cool.

After the grinding and polishing operations the lenses are parted from the pitch, and it is well known in the shops that the "figure" of a lens after parting from the pitch may be slightly different from the figure of the lens in the block: it is said to "come up a ring" or to "go down a ring," meaning that it shows a ring more or less after parting when tested by the Newton interference bands in the usual way. This effect is known as "springing," and its amount is slight so long as the pitch is in good condition. In part this alteration of figure is due to thermal expansion of the lens and pitch backing, the temperature developed by friction during the polishing operation being considerably above room temperature. This thermal expansion pushes out the centre of the lens and results in overworking of the centre, so that subsequently after parting off the lens has a hollow or even a distinct "hole" in the middle (Fig. 3, p. 128).

It is also a matter of common observation that the pitch in the crucibles becomes harder on account of the heating it receives, and by degrees the pitch becomes unsatisfactory, and then, apparently quite suddenly, it becomes very bad indeed.

The figures of lenses alter a great deal more on parting from the pitch, and a figure that was circular may become elliptical or three-cornered, or quite irregular. If the defective pitch is not replaced, another effect appears; after the first few minutes of polishing it may be observed that some of the lenses in a block are much more polished than others, the polished lenses being distributed irregularly among the rest. It may even be observed that a lens is well-polished on one side and comparatively grey on the other, and the grey edge may be oriented in any direction. This effect is known as "shrinkage" or "sinking off." With skilled workmen things do not reach this stage, but the symptoms of incipient "going off" on the part of the pitch are noticed in time to prevent more than a few lenses being lost. The pitch is replaced or new pitch is added to the old, and a more or less satisfactory mixture obtained once more. Now the pitch in the course of its distillation at the gas-works has been submitted for some time to temperatures of 350° C. and upwards. It is therefore a little surprising that any considerable alterations of its consistency should be brought about by further heating for a quite moderate time at temperatures but little above that of boiling water. Nevertheless at temperatures of 100°–150° C. the composition of the pitch changes, in the shops, quite rapidly: there distil away a number of gases representing the more volatile parts of the pitch formed by the gradual breaking up of more complex hydrocarbons, and in consequence there is a change in many of the properties of the pitch; increases are found in hardness, density, and free carbon content, while changes also occur in the coefficient of thermal expansion, calorific value, and solubility in organic liquids.

Section II. Physical Properties of Pitch and Alterations Produced by Heating.

Loss of Weight on Heating.

The percentage loss on heating at 150° C. was determined from two samples as follows:

No. 1 standard pitch of twisting point 38° C.		No. 2 pitch of twisting point 50°–55° C.	
Hours of baking	Percentage loss	Hours of baking	Percentage loss
34	2·8	31	2·63
65	3·75	61	3·9
95	4·6	86	7·65
120	7·43	101	8·7
135	7·81	146	8·85
180	8·00	170	9·12
204	8·90		

The samples were heated in evaporating dishes, and it was found that a skin formed on the top like cream on boiled milk: this film appears to hinder the free distilling off of the oils, and if it be ruptured a much more rapid evolution of fumes results. The irregular intervals in the percentages indicate more or less frequent rupturing of the film by stirring. The film may be due to oxidation or may be merely a more "de-gasified" layer.

Change of Density.

The loss of the oils is accompanied by an increase in density from 1·30 to 1·35 as shown in the accompanying table:

Standard Pitch

Hours of baking	Density	Hours of baking	Density
0	1·300	37	1·342
5	1·308	47	1·350
9	1·315	57	1·348
13	1·323	62	1·347
21	1·325	106	1·346
30	1·333		

In the later stages trouble with bubbles of gas in the pitch makes the density determinations too low; when two determinations differed slightly the higher value is recorded.

Chemical Changes.

The material lost on heating at 150° C. is evidently likely to alter considerably the chemical properties of the pitch; it may be expected to consist of the more volatile and more soluble constituents, that is, of the hydrogen-rich constituents, and there will be left behind the carbon-rich compounds or "free carbon." Tests were accordingly made of the No. 1 pitch to determine by ignition its free carbon content, and by Soxhlet extraction its solubility in CS_2 and other reagents. The figures obtained were as follow:

	Fresh	After 204 hours at 150° C.
Free carbon on ignition	47·8	53·4
Matter soluble in CS_2	63·9	58·7

Since the loss on heating for 204 hours was 8·9 per cent., the figures of the second column when corrected to refer to the original amount of pitch become:

Free carbon on ignition 48·7 %, an increase of ·9 %.
Matter soluble in CS_2 53·4 %, a decrease of 10·4 %.
Matter lost by distillation 8·9 %.

The figures suggest that the main effect of heating is to drive off part of the soluble matter; they also seem to indicate that other minor changes are taking place.

A pot containing 10 pounds of raw pitch was put out to bake under workshop conditions, and the solubilities of samples extracted at intervals were determined by the Soxhlet method to be as follow:

Hours of baking	Percentage solubility in CS_2
0	68
10	66·5
21½	66
30	65·5
51½	64
74	59·2
92	44·4

The change in composition appears to be greatly accelerated towards the end. While baking, the pitch was stirred at intervals to break up the skin previously mentioned and to facilitate the processes of evaporation.

A great number of tests have been made of the solubility and free carbon content of pitch as used in the shops, and when allowance has been made for the ash content, a very good idea of the quality of the pitch can be obtained from its Soxhlet figure. A systematic testing of the pitch by the works chemist would be a safe method for determining the point at which the pitch should be discarded, but the test is a lengthy process unsuitable for the workshops themselves, and better ones are now available.

When the fumes from the pitch pots in the shop are condensed on a cold plate they form drops of a thin oil which it seemed desirable to examine in greater detail. A retort containing some of the pitch was therefore enclosed in an oven at 150° C. and arrangements made to collect the distillate. Although a considerable amount of pitch was used practically nothing distilled over, and even occasional rupturing of the surface layer appeared to produce little effect. This seemed to indicate that the deterioration of pitch at the low temperature of 150° C. was contingent upon free access to the atmosphere. This would imply that the observed low temperature distillation in the open is destructive and not simply fractional, and that the oxygen of the air is responsible for the destructive action. It seemed important to determine whether any oxidation of the pitch could be observed, and for this purpose a variation of the usual apparatus for the combustion of organic materials was employed. The combustion tube was a few inches longer than the furnace, and the boat containing the pitch could be kept on the intake side of the furnace at a temperature of 150° C. by heating with a separate Bunsen flame regulated according to a mercury thermometer inserted through the rubber stopper closing that end of the tube. The vapours that distilled off were drawn into the furnace proper and oxidised at a bright red heat. The experiment was continued for about 12 hours, by which time the pitch had lost some 10 to 20 per cent. of its weight; the combustion tube was allowed to cool, the boat was removed and weighed, and after any oil that had condensed on the tube above the boat had been oxidised and carried through, the carbon and hydrogen contents of the volatile matter were determined.

In the two experiments made it was found that the carbon and hydrogen recovered by the absorption bulbs exceeded the loss in weight of the pitch in the boat, which must therefore have undergone some separate gain in weight. It is presumed that this indicates oxidation by the atmosphere. The results of the two experiments are given below:

Experiment (1).

Weight of new pitch taken	·398 gr.
Volatile matter distilled off (nett loss of weight)	11·8 per cent.
Actual loss of volatile matter (sum of hydrogen and carbon collected)	16·0 ,,
Hydrogen content of volatile matter	23·0 ,,
Carbon ,, ,,	77·0 ,,
Oxygen absorbed by original pitch	4·2 ,,
Oxygen in remaining pitch (in addition to its original oxygen)	4·7 ,,
Remaining pitch not combusted	

Experiment (2).

Weight of new pitch taken	·399 gr.	
Volatile matter driven off (nett loss of weight)	20·0 per cent.	
Actual loss of volatile matter	23·3	„
Hydrogen content of volatile matter	18·7	„
Carbon „ „	81·3	„
Oxygen absorbed by original pitch	3·3	„
Oxygen in remaining pitch (in addition to its original oxygen)	4·1	„
Hydrogen content of remaining pitch	3·96	„
Carbon „ „	90·5	„
Ash „ „	·94	„
Oxygen „ „	4·6	„

From these experiments it becomes clear that a considerable step forward could be made by enclosing the pitch vessel completely and protecting it from the influence of the atmosphere. The importance of this point is discussed later.

Hardness.

Probably the most striking feature of overheated pitch is the increase in its hardness. A quantity of pitch as used in the workshop (consisting of 10 parts of "standard" gas pitch and 4 of wood ash) was heated in the pitch mixing machine described later, at a temperature of 140° C. The mixing proceeded satisfactorily for some 60 hours; at 70 hours the pitch was stiffening rapidly; at 100 hours the machine could scarcely mix it, and the pitch had acquired a cindery character. If taken in the hot and liquid state—if liquid it could be called—and pulled out into a rope, the rope might be coiled up without coalescing into a heap. It was, however, by no means easy to pull out a rope to coil it without snapping it off.

The tests of hardness hitherto available have been of a very arbitrary nature. Apart from the tests by chewing, the determination of hardness has generally been settled by allowing a needle to sink into the material under a given load, and measuring the penetration in a given time. This method suffices to determine—subject to experimental errors of considerable amount in many cases—whether two pitches are of equal hardness, and if not, which is the softer. I am informed by Colonel Crompton that measurements on natural bitumens with pointed needles are untrustworthy, and that cylindrical needles with perfectly flat ends give the best results. This seems to agree with the experience of other investigators in respect also of coal tar pitch. But even the flat-ended needle is unsuitable when dealing with the much harder material used for lens backings.

In measuring the hardness of metals the most generally favoured method is to press a hard steel ball into a flat surface of the metal and to calculate the hardness as load in kilogrammes per spherical area of indentation in square millimetres. In order to standardise conditions a 10 mm. ball and a load of either 500 or 3000 kilos is usually specified. A somewhat similar method was adopted in our experiments for measuring the hardness of pitch. It will be shown in a separate paper that with soft wood pitch the indentation test is capable of determining a true physical constant of the material, viz. its viscosity, according to the formula

$$v = k_1 \frac{WTD^2}{d^4} \quad \text{or} \quad v = k_2 \frac{WT}{h\sqrt{hD}},$$

where W is the load, T the duration of the test, D the diameter of the ball, d the diameter of the indentation, h the depth of penetration from the original surface, and v the viscosity. The phenomena occurring when a steel ball is forced under great pressure into a backing pitch containing much solid matter are so different that it was found impossible to use these formulae even with modified indices; for the action appears to be discontinuous in many cases and the character of the depression—and not merely its size or shape—varies with the load and size of ball.

The edge of the depression is often marked by a series of minute cracks or crevasses, visible with a low power microscope. Similar cracks, closed up, may be observed inside the depression, as in Fig. 10, showing that the lip of the depression advances outwards partially by a flawing of the pitch just in advance of the loaded area. An empirical method was therefore adopted of using for workshop purposes a $\frac{3}{8}$″ ball, a 100 lbs. load, a temperature of 15° C., and a duration of test of 5 minutes. Under these conditions consistent results are obtained for the diameter of the depression, and the hardness is calculated according to the Brinell method as load per area of depression (kg./mm.²). According to this system of measurement, workshop pitches in good condition vary in hardness from 5 to 12

Fig. 10.

units; the softest varieties are used in general for the very large lenses, and the hard pitches for the small ones. A workshop pitch, kept in an open pitch pot, was found to begin life with a hardness of 9 units and to increase progressively to 22. A pitch kept continually stirred in an open mixer (*v.* p. 137) was found to increase from 7 units to 30 in four or five days.

Hardness of Pitch without Solid Matter, and Change on Heating.

The original pitch as received in the works gives a depression of the truly viscous type. The pitch of 38° C. twisting point gave with a load of 15 lbs. acting for 1½ minutes at 15° C. a depression of ·150″ diameter. After heating at 150° C. for 400 hours, it gave with the 100 lbs. load acting for 5 minutes at 15° C. a depression of ·075″ diameter—a Brinell hardness of 15. The alteration is striking. A pitch of 53° C. twisting point with a 15 lbs. load for 1½ minutes gave a diameter of ·105″. After heating at 150° C. for 170 hours in a small dish it was so hard that it always splintered under any considerable load, and no value for its hardness could be obtained.

Thermal Expansion.

There remains yet another effect of the prolonged heating, the last to be noticed here, and from the practical point of view an important one. This is the decrease in the coefficient of expansion, which makes itself felt by the behaviour of the pitch

in the refrigerator. A number of lenses were formed into standard pellets (*v.* Fig. 15) with good pitch, newly mixed, and an equal number with pitch just too bad for workshop use. They were placed together in a tray in the cold chamber at a temperature of 3° F. (– 16° C.). Within 4 minutes the good pitch showed signs of parting from the lenses, and all pellets were completely parted in just over 8 minutes. After three-quarters of an hour none of the bad ones showed signs of parting; it is probable that at this temperature separation would never take place. This difference can be used to give warning of approaching trouble before the polishing staff can observe it.

Section III. The Nature of Springing and Shrinkage and their Cause.

In order to study the nature of "shrinkage," a large number of blocks of lenses were made up and polished on the flat side. The lenses were plano-concave for the most part, 1″ in diameter and about ·08″ thick in the centre, the negative curve being 2·1″. The glass was extra dense flint, and there were 19 lenses in a block. After polishing, the block was left on a stand in the open room for a time, generally 48 hours, and it was then partially re-greyed on a flat tool. In consequence of the "sinking off" which had been going on during this time, the lenses no longer lay with their surfaces in a plane: some had sunk bodily below the general level, others had risen above it, while most had tilted and were half above and half below. In the photographs the parts that show grey are the high portions, and the polished parts are the low ones. The re-greying was carried out with the finest bruised "60 minute" emery, and the block was pushed about gently by hand over the tool, as care had to be taken not to overdo the greying. The photographs (Figs. 4–9) show a number of such blocks.

By the Newton interference test with monochromatic light it was found that shrinkage on a good block might be about 4 to 7 hundred-thousandths of an inch in 48 hours; with bad pitch it might be 10 times this. The amount of glass removed during polishing is generally one half to one ten-thousandth of an inch, varying slightly with the kind of glass and the character of the block, so that if a block has shrunk more than a few hundred-thousandths, it is hopeless to try to polish it without first re-greying. For this reason an effort is made in the workshop to polish a block immediately after smoothing. If the block has to be left overnight smoothed ready for polishing, it takes longer to polish next day, and if it is left over the week-end it is generally a waste of time to try to polish it. An improvement may be effected by bringing the block into intimate contact with a tool and leaving it so, when the lenses will not depart so much from the nominal contour of the block.

The knowledge of this fact and similar ones, coupled with the idea that pitch after all is a liquid, seems to have led to a belief that the slight sinking off observed in the shops was a sort of plastic flow under gravity; the pitch had only to be left long enough and it would let the lens sink down indefinitely. This is not the explanation, as was first shown by a Vicat needle penetrometer test. If a pellet of

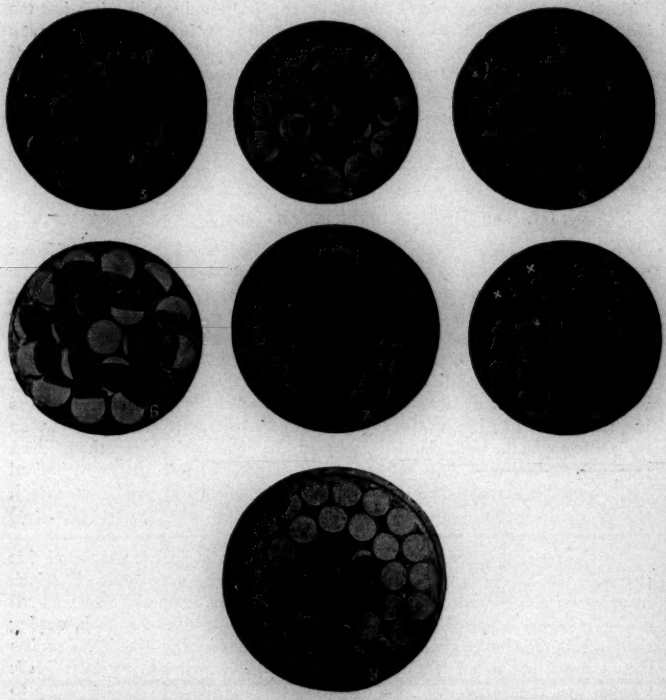

Fig. 3. Thermal expansion effect at commencement of polishing. The pitch contains 15% resin and the centres of the lenses are being pushed up by the heat of friction and polished first.

Fig. 4. Pitch containing 25% of resin, and considerable solid matter (about 30%). Block partially re-greyed after polishing.

Fig. 5. Pitch containing 50% by weight of solid matter (chiefly silica).

Fig. 6. Pitch containing no solid matter.

Fig. 7. A shop pitch from an open crucible. One night's shrinkage.

Fig. 8. A stiffer shop pitch. The shrinkage on the lenses marked with a cross is about 4/100,000".

Fig. 9. Effect of excessive heat in polishing. Note the tangential displacement of the lenses and the softening of the pitch at the centre of the block—the hottest part.

pitch under its own weight and the weight of a lens (say ½ oz. per sq. in.) can sometimes sink as much as 2 ten-thousandths of an inch in 18 hours, then under a stress of 300 lbs. per square inch, which was the load obtaining at the needle end, and which is 25,000 times as great, we should expect a penetration of the needle of 5″. The observed penetration was less than ·01″, even for a tolerably soft pitch, so that "sinking off" is not simply plastic flow under gravity.

It is, in fact, found that shrinkage may be treated under two heads, firstly, shrinkage that is merely thermal contraction, and secondly, plastic flow of the pitch under *internal strain*. Either type may originate at one of two points—(1) in cooling down after pelletting and blocking; (2) in cooling down after polishing.

1 (*a*). *Thermal Contraction in Blocking*. It is well known that in making a sensibly hemispherical block of lenses it is desirable to leave the block wrung home in the tool until it is quite cold, or, alternatively, to use a blocking tool of slightly different curvature from the smoothing tool, for thermal contraction will modify the curvature and make it difficult to bring up the outer edge of the block in the subsequent smoothing. The difference in radius of curvature depends on the thickness of pitch, is independent of the curvature, and commonly amounts to ·001″ to ·003″.

1 (*b*). The thermal contraction of pelletting and blocking sets up a certain amount of strain in the pitch, and in some cases this is sufficient to set up a "sinking off" during the early stages of polishing.

2 (*a*). *Thermal Contraction in Polishing*. The temperature of polishing is appreciably above that of the room. Tests made by pressing a sensitive thermocouple into the polisher immediately after removing it from the block seem to indicate that the average temperature of the top two or three hundredths of an inch of a pitch polisher is generally a little under blood-heat, about 30°–35° C. The temperature of the backing pitch is, of course, lower, depending on numerous factors such as size and character of lens, character of glass, arrangement of lenses, etc. But at temperatures of 25°–30° C. the hardness of backing pitch falls off very quickly, and there is a corresponding increase in the coefficient of thermal expansion. It is found that a block that is allowed to run very warm generally shows after cooling and re-greying a much greater shrinkage than one run at a more moderate heat.

2 (*b*). The thermal contraction after polishing may set up a certain amount of strain, and this will then produce a slight shrinkage of its own which may continue for some time.

The trouble under 1 (*b*) (strain set up by pelletting and blocking) is the most serious, and a pitch may be defined as good or bad in the workshop sense mainly by its behaviour under this heading.

The springing effect, as already mentioned, is in itself a proof of the existence of strain. The lens is obviously distorted when held by the pitch, consequently the pitch itself must be strained. The springing is slight, even with thin lenses, and its importance was overlooked for some time, until it was discovered that if a lens be separated at the contact of the pitch with the runner, the figure may be very different, both from that of the lens in the block and that of the lens parted from the

pitch. This seemed to show that the distribution of strain is more complex and more important than was originally supposed.

Now if there be strain in the lens we ought to be able to detect it in polarised light. A lens was therefore examined edgewise in a simple polariscope. Before pitching-on there was no double refraction visible; afterwards there was a good deal. A Babinet polariscope was therefore fitted up. If the instrument be illuminated with white light without any intervening specimen, the field is seen, of course, crossed by vertical lines, the central one black, the others coloured, and disposed symmetrically on either side of the black one. If a piece of good optical glass be placed in the specimen-gap no change is noticed unless the piece be stressed. A compression moves the bands bodily to the left, a tension moves them to the right, and a bending moment causes them to be inclined.

Now the amount of double refraction in an elastically strained body (i.e. the phase difference between the components of the light vibrating in the respective principal planes of stress) is proportional to the stress and to the thickness of the strained material through which the light passes. Consequently, by measuring the displacement of the central black line in the Babinet, we get a direct measurement of the stress in the material*. The values of the double refraction thus found are expressed in arbitrary units. 237 such units correspond to one wave length (sodium D line) relative retardation of the light components, and to 120 lbs./sq. in. compression or tension in a specimen 1″ thick in the direction of the light path.

We now return to the observations on the lenses themselves. The effect of mounting a lens on pitch will be either to bulge out or draw in the centre, putting the glass under a bending action. Examination of a number of lenses shows that in hand-made pellets the effect is to draw in the centre, while in the mechanically made pellets described later the canting of the bands in the polariscope indicates a bulging out of the centre.

It was expected naturally that a bad pitch would produce more strain than a good one, but it was soon found that this was not always the case. The reason probably is that, since the expansion of overheated pitch approximates more closely to that of glass, a higher stress in the pitch produces no higher stress in the lens. It was found, moreover, that the strain produced by "pitching-on" is commonly self-annealing at room temperatures; in fact, with good pitch, it is often found that within half an hour it cannot be detected at all. Now the time that elapses between blocking and the end of the polishing operation is not often less than two or three hours, and any springing that may be observed is due to residual strain remaining after that time. During the smoothing and polishing the pitch is rapidly annealing, its progress being no doubt materially helped by the heat generated in polishing. It is found, as might be expected, that the rate of annealing is faster for good pitch than for bad; and that whereas by the end of the polishing operation the residual strain of a good pitch may be negligible for workshop purposes, it may still be very appreciable with a bad one. The crucial test of a pitch, therefore, is its rate of

* Strictly speaking, this is only true of the case of a simple stress. If there be two principal stresses, the phase difference is proportional to the difference of the principal stresses.

annealing (relaxation period). A good pitch as a rule sets up strain as it cools, but little or no strain should be visible in an hour's time; the quality of a pitch may therefore be determined by inspection without making any measurements. It is merely necessary to examine a lens in the polariscope a few minutes after pitching-on, and again an hour later.

It might be suggested that the double refraction of the glass is due not to strain in the pitch but to the temperature gradient across the glass, which disappears as the pitch cools. That this is not so is proved by several considerations; firstly, that with bad pitch the strain may persist for weeks; secondly, that if the double refraction were due to the temperature gradient it would be greatest when the pitch is hottest; we find on the contrary that no double refraction appears until the pitch has been cooling for 5 or 6 minutes, and that the maximum double refraction is often not reached until a quarter of an hour, or more, after the pellet is made. The graphs (Fig. 11) are drawn from measurements of the strain in lenses after pelletting, showing the growth and decline of the stress with time. Pellet *A* annealed at a very slow rate; after reaching a double refraction of 300 units in 25 minutes it fell to 155 in 6 days and to 125 in 10 days. Pellet *C* was much like *A*, though it did not reach such an extreme maximum value. Pellets *D* and *E*, of good pitch, are typical, and may be compared with *F*, *G*, and *H* made on another occasion.

The curve is the net result of two components: (1) a curve of increasing stress proportional, more or less, to the amount of cooling the pitch has already undergone, and (2) an annealing of stress proportional to the amount of strain existing at the moment and to some function of the temperature. Expressed in non-mathematical language, we may say that the temperature of the cooling pellet follows a law corresponding somewhat to the lower curve in Fig. 12 *a*. If no annealing took place the strain could be represented by a very similar, or the same, curve, the amount of the strain being proportional, not to the ordinate, but to the distance *AB*, i.e., to the drop in temperature.

But this growing strain calls into play a slight plastic flow, or annealing. This plastic flow is slight, considered as a displacement, but a very little flow is sufficient to reduce very greatly the internal stress. At any instant the rate of flow depends on the strain at that moment existing, and the total amount of annealing that has taken place up to that moment may be represented by the length *AC*. Thus at time *t* a considerable amount of the fundamental strain has already disappeared, and there remains only a residual strain proportional to the difference between the two curves, i.e., to *BC*.

In Fig. 12 *b* the difference of the two curves—the residual strain—is plotted with the *t* axis as base, and we get the form of curve obtained experimentally. It will be realised, therefore, that the pitching-on of a lens may set in motion a much more complex series of phenomena than is commonly supposed though indeed it is nothing more than might be predicted from general scientific considerations. The practical significance of the phenomena depends on how long they endure in an aggravated form.

Returning to the phenomena that take place when a lens is pitched-on, we see

that complicated as they are already, the complexity is still more aggravated by the blocking operation. It is necessary for the proper examination of a specimen in the Babinet polariscope, that it should be as close to the quartz wedges as possible, so

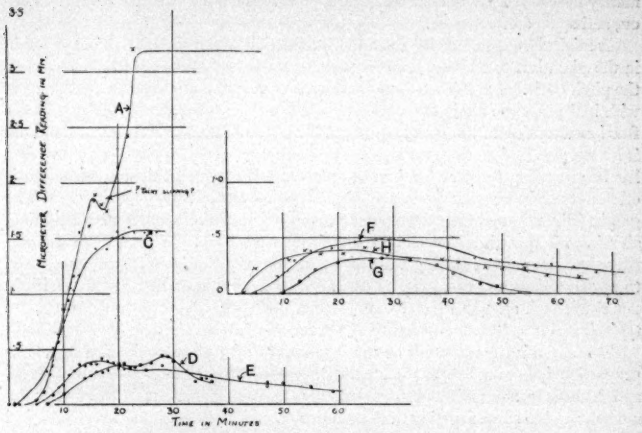

Fig. 11. Babinet tests of strain in pitch. Rise and fall of strain with time.

that it was impossible to examine workshop blocks; runners were therefore made no bigger than the pellets (1″ diameter) and a pellet carefully "blocked" which had become free from strain. The lines were found to be inclined sharply in the

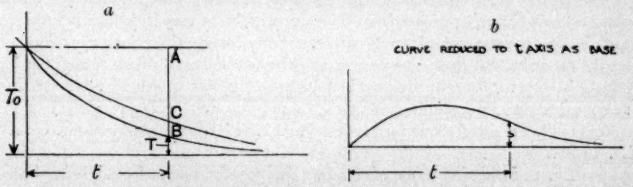

Fig. 12. Derivation of the strain-time curves.

opposite direction from that resulting from mechanical pelletting. In the first two specimens (using bad pitch) the micrometer difference readings were 400 and 220 respectively. The "run down" (i.e. the reduction in thickness of the pitch due to blocking) was only about $\frac{1}{100}$″ in each case—much less than is usual in the shops. Next a couple of pellets of reasonably good pitch were made, and in both cases

the slope was in the usual direction, and of values 133 and 125 at about the maximum point. The pellets were then "blocked" with 1″ runners, and the lines returned to the vertical. That is, *the effect of blocking was to cancel the distortion of the lens produced by pelletting*. On knocking off the runner again, the slope returned to its original direction, and was of the value 75 units, about half what it was initially.

We may therefore summarise the phenomena of pelletting and blocking with good pitch as follows:

Pelletting (by machine) bulges out the centre of the lens.

Blocking pulls in the centre and approximately compensates the pelletting.

The complex distribution of stress thus produced anneals during polishing, and the figure of the warm lens is the figure of a lens free from stress.

Thermal contraction of the pitch after polishing pulls in the centre of the lens*, and pitch which may anneal readily at polishing heat does not necessarily do so at room temperature. There is, therefore, a scientific basis for the workshop view that the figure of the warm lens in the block is the true figure. Such shrinkage as occurs with good pitch is due to the annealing of the strain set up by thermal contraction, and is worse the higher the temperature reached in polishing.

Mechanism of the Springing Action.

Consider a symmetrical pellet cooling (Fig. 13 *a*). The plane XX', being a plane of symmetry, remains plane however the general shape of the pellet may alter. The rule with cooling bodies which are chilled on the outside first, is that the form tends to approximate more closely to a sphere. The effect on our pellet will be to make its convex faces more convex. Now let there be a lens attached to one of these faces. The lens is pushed out at the centre as this face becomes more convex; it resists the distortion to the best of its ability and pushes out the plane XX'. The distribution of stress, in its simplest form, is indicated by the arrows, compression at the centre and tension at the edge (Fig. 13 *b*). Now let us "block" the pellet. The edge cools first and the subsequent contraction of the centre distorts the plane XX' still further, *but corrects the distortion of the lens*. The distribution of stress above XX' is tension at the centre, compression at the edge (Fig. 13 *c*). This stress complex now proceeds to anneal, and the final springing and shrinkage depend on which of the stresses remain after the polishing operation.

Numerical Value of the Surface Tractions.

It is a matter of some interest to determine what stresses must act across the contact of pitch and glass to produce a given distortion or a given double refraction. For example, what push in pounds weight does the pellet exert against the central portion of the lens to push it out one ring? The apparatus used to determine the magnitude of this push or pull is shown in Fig. 14. A lens ·08″ thick at the centre had a flat chamfer $\frac{1}{80}$″ wide ground on the edge of its concave side and by means of ozokerite this chamfer was cemented to a thick disc of glass of the same kind with a hole through the centre. This in turn was cemented to a metal disc with

* With double convex lenses the pitch will be thicker at the edges than at the centre, and then thermal contraction may pull down the edge of the lens.

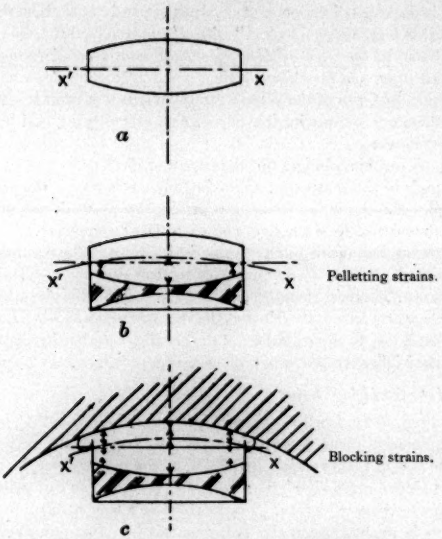

Fig. 13. Mechanism of the springing effect.

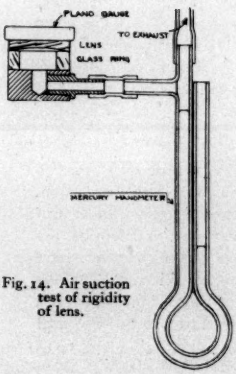

Fig. 14. Air suction test of rigidity of lens.

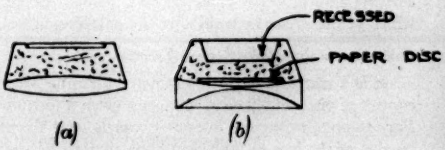

Fig. 15. (*a*) Coned pellet.
(*b*) Recessed and papered pellet.

arrangements for applying a suction by means of a vacuum pump. The vacuum was measured by means of a mercury manometer as shown. The lens was polished flat, and a contact gauge placed on the top, so that the hollowing that took place on applying suction could be observed.

It was found that with 14″ of mercury difference in the manometer tubes about 1¼ rings of hollowness appeared in the lens. These may be taken as hundred-thousandths of an inch deflection. The corresponding double refraction in the lens was ·27 mm. An inch of mercury corresponds to a pressure of ½ lb./sq. in., so that 1 lb./sq. in. suction on the lens would produce a micrometer difference reading of ·04 mm. and a deflection of two-millionths of an inch. The area of glass submitted to air suction was ·7 sq. in. so that to produce a double refraction of 80 units (say) the pitch must exert against the glass a pressure of $\frac{80}{4} \times \cdot 7 = 14$ lbs. at the centre, and a corresponding pull of 14 lbs. to balance this near the rim. The corresponding distortion will be 4×10^{-5}, that is, four black rings in the sodium light seen at normal incidence. It seems rather remarkable that most lenses experience forces of this magnitude at one point of their career, and that if the pitch be bad these pressures may be considerably exceeded.

There is a practice, occasionally adopted in the shops, of protecting the central part of a thin negative lens with a disc of paper or cardboard, so that the pitch only touches the glass on an annulus near the rim. This may be expected to reduce thermal expansion troubles somewhat, but the centre of the lens is not then pressed very evenly into contact with the polisher. An alternative method would be to recess the back of the pitch as in Fig. 15. The values of the two methods were tested by making up a number of pellets, some plain, some simply recessed, some fitted with paper discs, and some having both discs and recesses. They were prepared by the pellet-press machine described in Section IV under conditions otherwise identical. The results for three different pitches are given below, each figure being the average of five or six observations, which were themselves fairly consistent. The figures represent the double refraction produced in the lens and measured by the Babinet polariscope.

	Plain	Recessed	Papered	Papered and recessed
Good pitch 10 mins. after pelletting	80½	57½	47	19
Fairly good pitch 30 mins. after pelletting	30	18	18	2½
Bad pitch 10 mins. after pelletting	65	36	42	29

Although it is here shown that the presence of paper discs and of recesses tends to reduce the strains, both are to some extent complications, and both are unnecessary, except possibly for much larger and more flexible lenses than those described above, if the pitch is kept in good condition so that strain anneals rapidly. Papering is quite undesirable for several reasons, but recessing has some advantages, more particularly as a help to the mechanical handling of the pitch as described later. In practice a very slight recess is now used, which disappears in the blocking operation but facilitates pelletting.

There are two other causes of trouble sometimes experienced in the workshops when working lenses on pitch supports, which result in lenses remaining grey when

they should be polishing, and which are sometimes mistaken for the "shrinking off." They are, however, not strictly properties of the pitch at all. One is inadequate support. Imperfectly supported lenses naturally bend under the pressure of the polisher and consequently the polishing does not take effect. This trouble is easily recognised. The other trouble is a tangential displacement of the lens on its seating. This results in one part of the surface of the glass towards one part of the periphery polishing faster than the other. The lenses "hang grey" in a systematic manner, the grey edges of the various lenses being similarly oriented. It is easily shown mathematically that a displacement of less than a thousandth of an inch is sufficient to put one side of an average lens completely out of polishing range. It is well known that a block showing this defect may generally be completely polished by reversing the direction of rotation and so pushing the lens toward the other side of its seating. This is not a perfectly satisfactory method either for most rapid polishing or for best figuring. The tangential displacement can be completely eliminated by the use of proper pitch and proper "tacky" for cementing the pitch to the glass.

The Function of the Solid Matter.

It has been previously mentioned that it is a common practice to add to the prepared pitch a quantity of solid matter such as wood ash or ochre. Wood ash is the incombustible residue left from the burning of wood, and consists of mineral matter. It contains varying proportions of silica, alumina, iron, lime, magnesia, and potash. Its function appears to be two-fold. In the first place it reduces the thermal expansion and in the second it acts as a stiffening agent. The density of a heap of loose wood ash in air is ·7, and of wood ash in air well shaken together ·9, the density of the mineral matter itself being about 2·7. The interstices have, therefore, two or three times the volume of the mineral matter, and since the density of pitch is about 1·3, it would require pitch to the extent of 1 to 1·5 times the weight of the wood ash merely to fill the interstices. The actual ratio of the weight of pitch to that of the wood ash is 1·8 or rather more, which means that the grains of wood ash have generally a film of pitch between them, but only a thin film. It follows, since the elastic modulus of the ash is much higher than that of the pitch, that any load that comes on the mixture is transmitted mainly through the particles of wood ash, the function of the pitch being to prevent their lateral displacement. The load, however, has to be transmitted across numerous thin films of pitch, and these provide the facilities necessary for a slow yielding or annealing under continuous stress. The use of solid matter in this manner seems therefore to be a commendable practice, but the same cannot be said of the addition of resin to the pitch as a stiffening agent, a practice by no means rare.

Resin, as commonly marketed, is the resinous exudation of coniferous and other trees after the volatile part (oil of turpentine) has been largely driven off by heat. It appears definitely solid by ordinary tests, and although it has no definite melting point, the softening range appears to be much shorter than that of pitch or bitumen. The colophony of commerce seems to melt round about 105° C. Tested by the Brinell method, at room temperatures, the resin simply splinters under the load

without giving anything approximating to a Brinell impression, and for all practical purposes it may be said to be incapable of flow at room temperatures. A small addition of other materials, such as beeswax or pitch, makes a very noticeable difference to its behaviour, but in general a mixture of resin and pitch has a coefficient of expansion much higher than an equally hard mixture of pitch and wood ash (Figs. 3–6). The presence of more than 40 per cent. of resin in a pitch mixture makes it difficult to prepare backings which shall not crack away from the glass by thermal contraction to room temperatures, and mixtures containing much smaller amounts of resin are very much affected by temperature variations.

Section IV. The Mechanical Handling of Pitch.

In view of what has been said in the previous sections, it becomes possible to handle pitch mechanically with success. The pitch should first be brought to a standard hardness, as determined by the Brinell method, and as much solid matter as possible should then be added as is consistent with keeping the grains separated by a definite, though very thin, film of self-annealing material. The hardness of the mixture should again be tested and made to conform to the standard which experience shows to be necessary for the class of work on which it is to be used, and the polariscopic test should then indicate a definite strain 10 minutes after pelletting, but little or none at the end of an hour. Both hardness and strain tests must be made in a room at a steady temperature, and in the case of hardness testing, a standard temperature is desirable.

The pitch being satisfactorily prepared, the problem becomes one of keeping it from deteriorating and of handling it mechanically with precision and despatch. It is obvious that the method of "pitching-on" by hand precludes any possibility of making two pellets alike. The method is moreover slow and tedious, often hot and unpleasant, and in addition there is always the risk of a burn or two from the molten pitch.

The main requirements are three in number:

To avoid heating the pitch to a higher temperature than is absolutely necessary.

To keep the atmosphere away.

To keep the mixture homogeneous and prevent segregation of the solid matter.

The requirements are very satisfactorily met in the mixing machine shown in Fig. 16. The machine is totally enclosed, thus excluding the atmosphere*; it is heated by a steam jacket with steam at 40 lbs. pressure, thus maintaining a steady temperature of about 140° C. Continual stirring by two mixing blades as shown prevents segregation and keeps the mixture uniform throughout. In the example shown the trough holds some 20 to 30 lbs. of pitch and the blades rotate at about 15 and 18 r.p.m. This, of course, is a small machine for use with photographic and similar small lenses. Pitch is supplied to the mixture by throwing cold lumps into

* It is found that one of the quickest ways to harden pitch is to keep it in the open mixer continually stirred and with the free surface below the top level of the blades. By this means the Brinell hardness may be raised 1 unit every 2 hours. With the enclosed mixer 200 hours of heating and stirring failed to produce 1 unit of alteration.

the hopper *H*, where it melts and flows through a sieve *S* (to take out any foreign matter it may have picked up in the shops) into the trough, where it is stirred into the general mixture. The material is drawn out through the valve *V* on the end of the inclined delivery tube *T*, which is steam jacketted, so that the operator working the machine is completely screened from the heat and general obnoxiousness of a room containing molten pitch.

When the pitch is delivered into the other room it may be used for forming layers or pellets as required. As the best work can generally be done with pellets it will suffice to describe the methods of preparing pellets. Examination of existing workshop practice showed that the thickness of hand-made pellets was not proportional to the diameter of the lens. A fair empirical relation was found to be

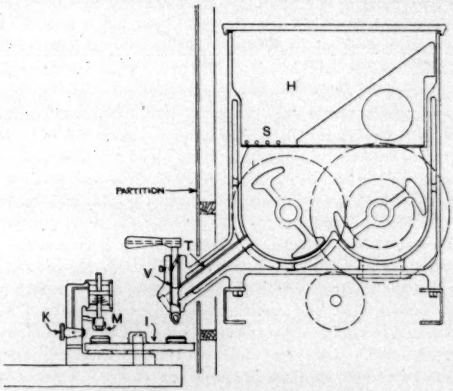

Fig. 16. Pitch mixing and pelletting machine.

$t = \cdot 3\sqrt{d}$, t and d being both in inches. This rule was adopted with satisfaction to standardise the thickness of mechanically prepared pellets. For a number of reasons pellets should not be cylindrical, but coned as shown. This is not only an advantage in blocking (Fig. 17), but the draught is useful in separating pellets from their moulds as described later. The standard (double) cone angle adopted for all negative and most positive lenses is 35°. For exceptionally sharp-radiused crowns 60° is used.

A simple form of pellet press as used for experimental work is shown in Fig. 16. The lens is carried in a cradle (Fig. 18) and covered with tapered mould. Below the delivery tube of the mixing machine (Fig. 16) is a rotating table *I*, with a number of stations fitted with cradles containing the lenses as shown. The pitch flows continuously from the nozzle of the delivery tube, and as each mould is filled the table is indexed on by one station. After passing through several stations, during

which time the pitch is cooling to a plastic state, the mould reaches a station under a pneumatic press. At this point a turn of the knurled wheel *K* brings down the water-cooled die *M*, which forms the upper surface of the pellet, and squeezes out the superfluous pitch which remains attached to the pellet by a thin film. At the next station the mould and pellet without the cradle are transferred together to the separating machine, where they cool, and then come under a hammer which separates the pellet from the mould, and sends it down a conveyer to a collecting point, while the moulds travel down a chute to another point. The pellets thus made are of a uniform shape and are made very rapidly and with very little trouble or unpleasantness to the operator. The process is largely automatic and is more so in more recent machines.

A more complete account of some of the mechanical features of the machines employed will be found in British Patent Specification 141,395 (or American Specifications 1,332,778, or 1,358,676 and 1,358,677) which should also be consulted for information concerning the patented features of the principles involved.

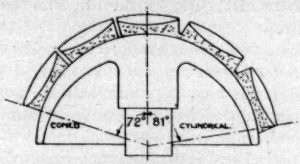

Fig. 17. Value of coned pellets in deep positive blocks.

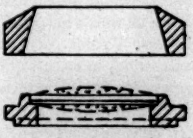

Fig. 18. Coned ring and cradle (lens shown dotted).

Concluding Notes on the Use of Pitch for Optical Work.

Any method of mounting lenses in blocks must satisfy the following requirements:

(1) It must hold the lenses firmly against normal or tangential displacement.

(2) It must leave the lenses free from stress.

(3) It must be able to accommodate slight variations in the diameter or thickness of lenses nominally alike.

(4) It should be applicable to positive lenses with knife edges or to negatives with very flexible centres.

(5) It should be simple and clean, and adapted to mechanical handling.

Obviously no mechanical chuck or similar device can satisfy these requirements.

Pitch as a material for mounting lenses for polishing possesses many remarkable advantages. Its colour is valuable; the dull black surface in contact with the lens prevents reflection of light at the second face of the glass, and nothing better could be devised for the purpose of examining figure in a block. More than most similar materials in the "solid" state its coefficient of expansion approximates to that of glass, it melts at a relatively low temperature, and remains plastic through a considerable range. A material with a definite melting point would be much more

difficult to handle, and in the solid phase would need a coefficient of expansion almost identical with that of the glass to be worked, or figure would always be bad. Even if this condition were satisfied, the material would be at a disadvantage as compared with pitch, since identity of expansions would preclude the use of refrigeration for parting Pitch, being an undercooled liquid, i.e. a viscous material, may be made sufficiently solid to resist deformation by external pressures during the polishing operation, and yet left sufficiently plastic to yield to internal stresses, so as to be self-annealing at ordinary temperatures. The alteration of properties on prolonged heating is its most serious disadvantage, but the methods of conservation and testing described above seem to be capable of dealing with this, and of utilising to great advantage the unique properties which it certainly possesses.

DISCUSSION.

Dr J. W. French: While pitch has many vices, it has also many virtues, but probably the latter are more apparent to practical opticians, few of whom have yet succeeded in finding a suitable substitute.

We are all much indebted to Mr Preston for having given us the results of his experiments. To this particular subject the pioneer opticians must have devoted much attention, but until quite recently it has been assumed that there was but little scope for further research work. In the past reliance appears to have been placed upon the earlier investigations and, considering how much research work has been done in other branches of the optical arts, it is somewhat remarkable that pitch should have been so long neglected as a subject of research. It is not suggested, however, that the importance of pitch control has not been realised or neglected in well-directed establishments: it is the laboratory aspect that appears to have been overlooked.

Personally the habits of a lifetime make it extremely difficult for me to regard pitch, even when mixed with 30 per cent. of wood ash, as a substance whose viscous properties are of secondary importance, as Mr Preston appears to do. I cannot forget with what enthusiasm Lord Kelvin in his Natural Philosophy classes used to describe to us the viscous properties of pitch which he regarded as a most delightful substance. Each session he placed on the surface of a dish full of pitch bodies of various forms such as tetrahedrons and from time to time he showed us how these bodies assumed their positions of stable equilibrium and how corks placed at the bottom rose in the course of time through the pitch to the surface, just as they would in an incomparably shorter time float to the surface of water.

I prefer, therefore, to refer the troubles that may be experienced in the use of pitch to its property of viscosity and this view is, I think, justified by the results of the following experiment. I obtained from the workshop two pieces of sticking-on pitch in a condition for immediate use. Each piece was attached to a stout glass disc of one square inch area and the pieces were laid down side by side upon a flat steel plate. The one was loaded with half a pound and the other with five pounds. Measurements were taken at regular intervals. The results are as follow:

Both specimens sank rapidly during the first few hours; after settling down they sank more slowly and fairly regularly during 25 hours, but the small-load specimen sank at less than half the rate of the high-load specimen. After 25 hours the rate of sinking decreased and at 50 hours the rate in the case of the small-load specimen was practically negligible, whereas in the case of the high-load specimen sinking continued, although at a comparatively slow rate, for more than 100 hours during which this particular test was made. In view of the fact that the pitch was combined with 30 per cent. of wood-ash, a difference in the rates of much more than 2:1 is not to be expected, although the ratio of the loads was 10:1.

It is interesting to consider why, after about 50 hours, the rate of sinking has so greatly decreased. It is easy to see that the exposed surface of the pitch has become very hard and brittle, due, I think, to the evaporation of solvents, although I quite recognise that in the case of resins surface oxidation probably occurs. After about 50 hours I believe the outer skin of the pieces of pitch has become so hard as practically to form a case which holds the more viscous interior pitch and this outer case prevents deformation when the load is light. When, however, the load is heavier, deformation of the outer shell at a comparatively slow rate appears to continue for a much longer time. It is only necessary after 50 hours to spray very lightly with a solvent the surfaces of the pitch to realise how erratic the action then becomes. Portions of the casing become soft and allow the pitch to flow irregularly. Similar interesting results can be obtained by covering the specimens with a bell jar containing a small amount of naphtha vapour.

If Mr Preston regards the property of viscosity of workshop pitch as being of very secondary importance, as he appears to do, it is then necessary to attribute the variations observed in the workshop to some such condition as defective annealing. But, if workshop pitch is the viscous material which I believe it to be, then it is hardly possible for such stresses to exist. Workshop evidence, such for example as the direction of sinking of lenses on a spherical head, all points to the pitch having a considerable amount of viscosity.

With regard to the oxidation of resinous substances, it may interest the Members to know that the reddening of balsam by oxidation that takes place when it is being evaporated to a solid form can be overcome by evaporating the balsam in a vacuum. The solid balsam thus obtained retains its original yellow colour.

Mr P. F. Everitt: I remember how in one optical firm a number of lenses mounted on pitch were left for a long time and had ultimately to be knocked off and re-mounted before working. The modern tendency is to adopt methods of mounting without pitch pellets. In course of time many of the problems connected with pitch will disappear. The author's treatment of the subject is not complete, but it forms a good starting point for other investigators.

Mr Preston: It is one of the axioms of my paper that pitch is a "viscous fluid." The properties I have described are only possible in virtue of the fluidity thus postulated: they could not exist in a rigid substance and the last thing therefore that I want to do is to suggest that the fluid nature of pitch is a matter of secondary importance. On the contrary, it is the keynote of the whole paper.

There is some confusion in Dr French's use of the term "viscosity," especially when used in a quantitative sense. Thus he says "The evidence all points to the pitch having a considerable amount of viscosity," when it is meant that it has a considerable amount of fluidity, or lack of viscosity. For the more considerable the amount of viscosity, the greater the rigidity of the substance and the greater are the stresses it can carry. All the experiments of Lord Kelvin and others are designed to show, not that pitch is *viscous*, but that it is *fluid*. Once its fluidity is established, it is self-evident that the fluid is a highly viscous one.

Now although pitch is a fluid, its viscosity is so high that it can support enormous stresses without rapid flow. A heavy weight may be placed on a lump of backing pitch, and the pitch is thereby subjected to considerable stress; but it does not instantly flow away and let the weight down with a bump. Thus it is perfectly obvious that, though workshop pitch is the viscous (more strictly, fluid) substance that Dr French believes it to be, yet it is entirely possible for great stresses to exist in it.

The paper is intended to lay emphasis on the importance of those stresses that originate internally from thermal expansion and contraction, as distinct from stresses arising from external loads. These internally-arising stresses can be borne for a time by the pitch, just as heavy external loads can be supported for a while. But inasmuch as pitch is a fluid, it forthwith engages on a slight differential flow —viscous flow—to relieve these internal strains. This is the process of *annealing* and is identical with the process of annealing in glass.

The internal strains may be initially high—they are higher, the greater the viscosity of the pitch—but they are entirely relieved by a very small viscous displacement, so that flow under internal strain does not continue indefinitely. As soon as annealing is complete, the flow stops. Under external loads, on the contrary, the load is not removed by the flow, and flow may continue indefinitely (in certain cases buoyancy and surface tension may bring it to a stop).

We see, then, that so far from defective annealing proving that pitch is not a viscous material, it merely proves that pitch is a very viscous material. My results do not contradict Lord Kelvin's, but they could be predicted from them.

Dr French's experiments indicating that a semi-rigid casing forms on the outside of the pitch even at room temperature is interesting. It seems entirely possible and even probable, but whether it is thick enough to be a factor of great importance would seem to require further experimenting.

I am glad that Mr Everitt thinks that many of the problems connected with pitch will ultimately disappear: but I do not think they will disappear spontaneously. We have a long way to go and much thought will need to be given in the future to the details of our workshop problems.

OPTICAL DESIGNING AS AN ART

By H. DENNIS TAYLOR

MS. received, 24th February, 1923. *Read and discussed, 8th March*, 1923.

ABSTRACT. The chief aim of this paper is to show that the tendency, which has been developing among optical designers in recent years, to follow in the footsteps of (mostly) German optical workers and test the performance of optical designs by exact ray tracings by trigonometrical methods on paper instead of by inspection of the actual performance of the embodied idea, involves a needless waste of both time and working costs. While such exact ray tracings may give valuable data concerning the optical performance of a lens system, if carried out thoroughly enough, yet the results so obtained do not and cannot give any indication of the direction in which changes are required, and only an appeal to the general algebraic formulae, such as those, for instance, founded upon Coddington's work, can afford the designer the enlightenment necessary to enable him to bring the system to the best possible perfection. Therefore, I have fully come to the conclusion that ray tracing methods are, generally speaking, unnecessary and superfluous for any designer who cares to master the general implications of the much more philosophical and illuminating system of optics founded on Coddington and other workers of the British school. This conclusion I seek to illustrate and justify by a few illustrative examples from my own experience.

Our ex-President, Professor Cheshire, recently made some suggestions to me as to the desirability of my contributing a paper (even though a belated one) upon the question of "The future of Geometrical Optics," which has been a very vexed question since the convention which took place at Cambridge in May 1921 and which I was not able to attend. I have read the report* of the addresses and discussions at that convention with great interest, and was much impressed by the opening address by Commander T. Y. Baker in which he largely dwelt upon the quantitative inaccuracy of the algebraic formulae of the second order for the four main aberrations, especially under certain conditions, thus to my mind emphasising the call that there is for the further development of the formulae of the third order. In fact it is fairly well known to us that the algebraic formulae at present available are often more valuable in their qualitative sense than in their quantitative sense, and, as Mr Conrad Beck well observed, "a mathematical formula may be of no practical use for direct application, but may be of the utmost value in revealing tendencies."

Then, as bearing upon the comparative merits of algebraic systems of formulae of the second order on the one hand and the more exact procedure of ray tracing on the other hand, almost as if they were rival processes, one of which might supplant the other, we find that Professor Cheshire cited the case of the firm of Messrs Carl Zeiss for whom Professor Abbe had inaugurated the rigid rule that

* *Trans. Opt. Soc.* 22 (1920–21) 201.

the designing and computing of optical systems must be completed in the computing office without any recourse to workshop trials. This matter may be found further enlarged upon in the letters of Professor Abbe to a friend in England, recently published by Professor Cheshire, and I shall have occasion to refer to it again.

Throughout this paper I shall speak of the several orders of formulae as follows:

Formulae of the first order. These comprise the Gaussian formulae, or the formulae of narrow pencils of disappearing apertures, in which no aberrations whatever are assumed to have a place, and therefore represent the ideal performance to which perfectly corrected lens systems of even large fields and apertures shall conform.

Formulae of the second order. These embrace the usual algebraic formulae for:

(1) Spherical aberration.
(2) Curvature of field and eccentricity corrections.
(3) Coma.
(4) Distortion.

Formulae of the third order. These involve higher developments of the reasoning by which the second order formulae have been arrived at. As yet these formulae have only been worked out to a very small extent.

I should say at the outset that the position which I shall attempt to justify is this, that it is the Algebraic System of Optics of the first and second orders of the British school as initiated by Sir George Airy and Henry Coddington, and also the undeveloped third order of the same, which together constitute the source of creative inspiration by which all advances in optical constructions and inventions are rendered possible, while the practice of what is known as ray tracing, so largely employed in Germany, should only be regarded as of quite secondary importance, so much so, in fact, that its employment is quite unnecessary, provided that the algebraic system and all its implications be sufficiently grasped and digested. Personally I substantially agree with a remark made by Mr Horace Beck at the Cambridge discussion—"The present method of trigonometrical calculation is nothing more than the old fashioned rule of thumb method of trial and error. In lenses where it is easy to measure the aberrations accurately, it is much more rapid and cheaper and quite as satisfactory to measure the results and by tabulating and examining the aberrations, methods of eliminating them can be found."

I should say that, with the exception of the accurate trigonometrical tracing of marginal rays entering large telescope object glasses parallel to the axis, I have never done a particle of ray tracing, nor do I intend to do so without being more convinced than I am yet of its necessity, or even of its desirability. In fact, if I had undertaken as many ray tracings as would have been called for in the course of working out the many new optical combinations, of which I am dealing with a few samples in this paper, not only would the combinations never have been completed, but I have grave doubts as to whether I should have been alive and addressing you to-night. As the devising of these new optical combinations and the evolution of a system of algebraic formulae of the second order have gone on more or less in close inter-relation and simultaneously, I will here give a short

sketch of the experiences which have led me to take up the attitude that I do on the pressing question of whether this country is to develop and work its own optical system, or whether another much more mechanical system shall be imposed upon it from without. About the year 1890, after carefully studying Coddington's *Optics*, I entered on a course of development based upon his system and notation, which led to formulae full of important suggestions. The possibility of correcting the curvature of field and oblique astigmatism of a positive lens, by placing on the same axis another negative lens of the same power and of the same glasses, and trusting to the axial separation for obtaining sufficient positive or image forming power for the combination, at once occurred to me, and led me to make a series of experiments from which I soon learnt that rectilinear images could not easily be obtained without dividing the positive lens into two positive lenses equal or unequal in power and placing the negative lens between them. By adopting that course and employing my formulae just in a broad sketchy manner in which no allowance was made for lens thicknesses, I made a model which formed the basis of the achromatic or compound triplet lens illustrated in Fig. 1. The method was first to make the sketch model, then measure its deviations from the ideal performance, and the changes in its curvatures necessary for eliminating the errors were then inferred in a general way from the formulae, and so on until the finally approved standard was reached, when the result was a flat field anastigmat lens of F/8 aperture covering a plate of length about equal to the focal length. But you will see that I was then labouring under the idea that each of three lenses should be self-achromatic and self-corrected against spherical aberration as well as free from coma. It was not until some time after that it occurred to me that after all an achromatic final image free from spherical aberration could be obtained even with three quite simple lenses provided their separations bore the proper relationship to their powers. Therefore triplets of three simple lenses soon followed with results as good as, if not better than, those given by the first more complex lens, and further, as the new Schott barium crown glasses of high refractive index with low dispersion and highly dispersive flints combining a comparatively low refractive index with a comparatively high dispersion were then coming into the market, I was able to devise new triple combinations embodying the new glasses and utilizing their optical benefits. So far the first trial model of any of these lenses had been the embodiment of a very rough sketch whose performance fell very far short of the final performance, requiring very many tentative alterations in relative powers, positions, and curvatures, all such being suggested by appealing to the formulae and carrying out whatever they pointed to in the way of alterations. Later on, however, I began to work out the first model for new photographic

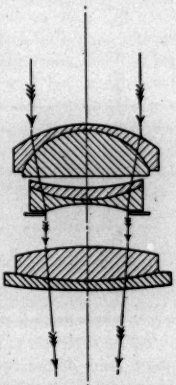

Fig. 1.

lenses rather more elaborately, with, I think, some saving of time, and this became more desirable as the number of simple lenses in a combination was increased for some reason or another to more than three.

For instance, in Figs. 2 and 3, the idea aimed at is a triple anastigmat flat field lens of about F/4 full aperture corrected and designed to perform at its best not with the stop at or near the middle of the combination, but at a distance outside of the first principal point equal to 1/3 of the effective focal length. It is wanted for a special purpose. As in all the later triple Cooke lenses, the first

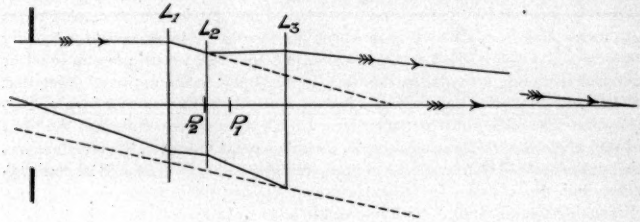

Fig. 2.

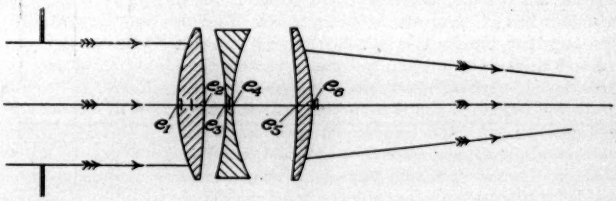

Fig. 3.

procedure is to settle the glasses to be employed, and next to determine how closely it will be desirable to fulfil the Petzval condition, and then to fix the ratio between the powers of the two positive lenses. This being done and the glasses being known, the next thing is to settle the relative axial separations between the lenses which are necessary to secure both the required achromatism (for rays C and F in this case) at the principal axial focus and also similar achromatism of the oblique focal points so that the dimensions of the images in the two colours (C and F) shall be equal. For this purpose the lenses are at first treated as infinitely thin, as mere abstractions. The required separations having been settled, the next procedure is to work out the parameters α, β, and x for the three lenses, α characterising the state of vergency for rays of axial pencils for each lens, β being the same characteristic or parameter for principal rays of oblique pencils crossing the axis

at the centre of the stop, and x being the parameter or characteristic of the shape of each lens; also the heights y from the axis at which the marginal rays of an axial pencil of parallel rays impinge upon each lens plane, and the heights Y from the axis at which *principal* rays through the stop centre impinge upon the several lens planes. Then the formulae for each lens can be worked out by four figure logs expressing:

(1) The spherical aberration for a pencil of parallel rays entering L_1 at height y_1 and referring the aberrations of L_1 and L_2 to the position of L_3.

(2) The eccentricity corrections affecting the focal lengths of oblique pencils in primary planes for each of the three lenses, in which β is involved.

(2*a*) The sum of the curvatures of field of the three lenses combined for central oblique refraction, ascertained both in the primary and secondary planes. Then the eccentricity corrections of course are ultimately required to be minus and equal to this curvature sum, at any rate in primary planes.

(3) The angular coma affecting the foci of oblique pencils for each lens. These should sum up to zero.

(4) The distortion of straight lines in the images yielded by each lens. These should sum up to zero.

Then having got all four formulae for each lens in the form $Ax^2 + Bx + C$, the next thing required is to make a table of numerical values for a few different values of x centering round what is considered to be a likely value for it. This being done, one can pick out the values x_1, x_2, and x_3 for each lens which yield the best general corrections (1) for axial spherical aberration, (2) for flatness of field in primary planes, (3) for freedom from coma in the oblique pencils, and (4) for freedom from distortion of straight lines.

As an instance Table I gives the initial data employed for the system shown in Fig. 3.

Table I.

Lens	Nominal focal length*	Radii of curvature	μ_D	ν	Axial air-separation
L_1	+ 2·67	+ 1·956 + 9·78	1·61	58·8	·30 approx.
L_2	− 2·03	− 3·216 − 1·982	1·604	38	·72 approx.
L_3	+ 4·96	− 13·58 + 2·495	1·61	58·8	

[* i.e. $(\mu - 1)(1/r_1 + 1/r_2)$.]

The effective focal length of the combination is about 7·1 inches, but this quantity is not determined or measured accurately until the performance has been finally approved; then the curves for any given effective focal length are just a matter of proportion. This combination is not yet fully worked out. Table II gives the corrections obtained for the initial model.

The spherical aberration is about − ·036 linear for an aperture of F/4·78, the image is almost flat in primary planes, the coma amounts to only about 40 secs. angular value at 18° from the axis, while the distortion is only ·44 per cent. of the radial dimensions of the image at 18° from the axis.

Table II.

Lens	x value	Spherical aberration	Eccentricity corrections, P.P.*	Angular coma, P.P.	Distortion
L_1	+ ·66	+ ·03638	+ ·06870 $\tan^2\phi$	− ·02651 $\tan\phi$	− ·1138
L_2	− ·2375	− ·04760	+ ·00216 ,,	+ ·03803 ,,	+ ·1732
L_3	− 1·45	+ ·01216	− ·22290 ,,	− ·01218 ,,	− ·0550
Combined total		+ ·00094	− ·152 ,,	− ·00066 ,,	+ ·0044

Linear aberration = − ·036. Curvature, P.P. = + ·1510. Angular coma = 40″.

[* i.e. primary plane.]

Of course these results are only rough approximations; we are dealing with a mere skeleton arrangement of the finished lens. The next step is to translate the three lens planes of Fig. 2 into three lenses of the required aperture shown in Fig. 3. Then instead of three lens planes we have six element planes e_1 to e_6, being planes tangent to the six lens surfaces, and if the three thicknesses are t_1, t_2, and t_3, we may consider the air separation between e_1 and e_2 to be t_1/μ and so on. Then if it be desired to work out the combination on paper more accurately, these six elements must first

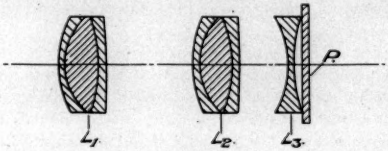

Fig. 4.

be distributed again as to separations in order to fulfil the conditions of axial and oblique achromatism and then the four formulae for each of the six elements be worked out anew, when the results given in the last table may be found more or less modified so that a fresh adjustment of the three x's may be found desirable. This second approximation consequent upon the introduction of concrete thicknesses is the most desirable when one or more of the positive lenses are of meniscus form having x much greater than 1. However, whether the skeleton form be a first or second approximation, the next step is the practical one of getting the lens carried out in the workshop according to Fig. 3, when it may be mounted in a trial camera and its various errors be measured up. Then an inspection of the tables of formulae for the four corrections for each of the three lenses will enable the designer to modify the x's of the lenses or the powers of the lenses so as to bring about the various differential corrections required to annul the observed errors. These having been carried out, the lens is again examined and so on until the designer is satisfied that the very best compromise of corrections has been obtained.

We will now proceed to a more complex case of a photographic lens as shown in Fig. 4. The intention here was to produce a flat field rectilinear anastigmat of the very large aperture of F/2, chiefly for cinema photography, and in order to secure this very short focus relatively to aperture the negative corrector lens was

placed as close up to the focal plane as possible instead of being placed between the two positive lenses as in the last case. It is clear that its influence for decreasing the power of the combination is then at its minimum. It now becomes essential that the two positive lenses shall each be about self achromatic or very slightly undercorrected. In order to get the four formulae for each compound cemented lens simplified down into the form $Ax^2 + Bx + C$, it is necessary that the x shall refer to the centre positive lens only, since the three components are to be fitted together for cementing. It is then perfectly simple to express x_1 for the first component and x_3 for the third component both in terms of x_2 for the second component.

Then, as in the last case, each compound lens is treated as all lying in one plane, again a mere abstraction, and the four formulae for each are worked out numerically for different likely values of x_2 and x_5, after which a suitable selection is made of such values of these as well as x_7 (for the negative lens) as will yield the most satisfactory general result. Should this not be forthcoming, different types of glasses or different separations are considered until the desired skeleton result is obtained after which, without in this case troubling to work out the corrections for the elements constituting the lenses of concrete thicknesses, a model of 2 to $2\frac{1}{2}$ inches effective focal length was constructed and its errors measured up. It was soon found that, as was expected, the axial pencil when free from spherical aberration in a general sense yet showed a marked zone of aberration, but not enough to mar the definition except under high magnification. This lens had to go through many trials and alterations before the final result was obtained, the field of view being a circle of 30° diameter so that a focal length of 2·25 inches would just cover a cinema film picture to the corners.

Turning now to Fig. 5 we have another quite different type of optical construction. It represents the very simplest form of unit power and wide angle periscope, and consists of two simple negative lenses L_1 and L_2 for the first or top objective, a reflector m_1, a large achromatic objective O, half-way down the long tube, and a second reflector m_2, a glass diaphragm or field lens L_3, and two simple positive lenses, eye lenses L_4 and L_5, the whole giving a flat achromatic and anastigmatic field of 60°. P_1 is the entrance pupil and P_2 the exit pupil. This simple anastigmatic periscope was worked out upon much the same lines as the photographic lenses.

This basic construction, together with a different method of erecting the image, was first worked out, in response to the urgent representations of officials of the Royal Air Force, for the purpose of enabling the pilot of an aeroplane to get a clear view of the ground below and in front of him. After being worked out and submitted to the Government Air Experimental Force the instrument was kept six months and then returned to us without even having been looked through, so far as we could ascertain. I simply cite this as an instance of the very casual manner in which inventors are often treated by Government Departments.

As to submarine applications, we are all only too painfully aware that the great restrictions upon the calibre of the tubes used for submarine periscopes renders the attainment of such an ideal construction quite out of the question, so

that the present clumsy and complicated system of positive lenses, constituting a veritable outrage in optical design, and absorbing at least seven-eighths of the entering light, has had to be adopted.

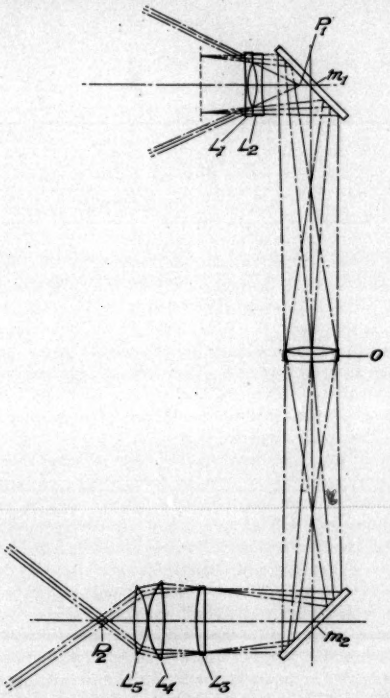

Fig. 5.

One feels disposed to ask, while on this subject, whether the conflicting requirements of the utmost possible invisibility of the submarine periscope to the enemy on the one hand, and the utmost possible visibility of the enemy ship to the submarine on the other hand, have been compromised to the best advantage, or whether the simplification of the periscope and its conforming better to sound optical principles, and the consequent great saving of light and improvement in definition away from the centre of the field, would not more than balance the

increased calibre and visibility of the tubes that would be required. Even if this matter is considered as settled, it might still be quite feasible for a submarine to carry a simple and effective large calibre periscope for use in picking up very distant enemy vessels, and resort to the present small calibre periscope when approaching near enough to the enemy ship to run the risk of being detected.

And now we may, with special advantage, consider the question of the microscope, for it is with regard to the higher power microscope objective of large numerical aperture that the strongest arguments in favour of the methods of ray tracing may be brought forward.

When I last read a paper before this Society, I said I hoped on some future occasion to introduce to your notice a form of anastigmatic flat field microscope.

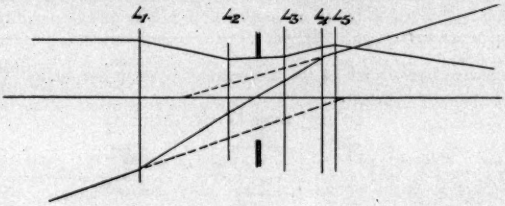

Fig. 6.

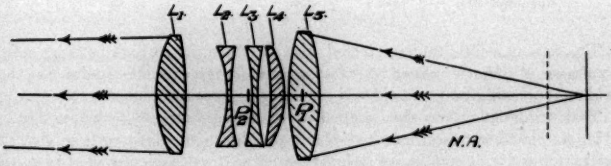

Fig. 7.

Fig. 6 is the skeleton arrangement for an anastigmat objective intended for about 1 inch focal length or so with an N.A. of about ·225. This consists of three positive lenses L_1, L_4, and L_5 and two negative lenses L_2 and L_3 instead of the two positive and one negative as employed for the photographic objectives, which latter were intended for an aperture of F/4·5 only whereas for our micro-objective we want nearly F/2. Moreover we are up against a new problem. The triple photographic lenses all show at full aperture a short or positive zone of aberration, which I do not consider can be eliminated, but it is not bad enough to be detrimental. But the same zone in a micro-objective would be very detrimental to the definition, as the image is further magnified up by the eyepiece. Therefore one of our new conditions is freedom from zones of aberration, and to that end alone it is desirable to have three positive lenses instead of two, as will be better realised later. And

if there are three positive lenses there must be two negative lenses, for the negative aberration of one negative lens will be far too strong for the three positive lenses.

Table III gives the initial data of the system shown in Fig. 7 in its original form.

Table III.

Lens	Nominal focal length	Radii of curvature	μ_D	ν	Thickness	Axial air-separation
L_1	+ 2·6	+ 1·748 + 12·24	1·6105	58·8	·35	·96
L_2	− 1·734	− 1·491 − 4·473	1·6457	34·0	·08	·63
L_3	− 1·734	− 2·033 − 2·485	1·6457	34·0	·08	·325
L_4	+ 3·9	− 2·294 + 1·147	1·6105	58·8	·22	·005
L_5	+ 1·95	+ 2·415 + 2·185	1·6105	58·8	·35	

The effective focal length of the combination is about 3·2 inches. Table IV gives the corrections obtained.

Table IV.

Lens	x value	Spherical aberration	Eccentricity corrections, P.P.	Angular coma, P.P.	Distortion
L_1	+ ·75	+ ·01758	+ ·5595 $\tan^2\phi$	+ ·1161 $\tan\phi$	+ ·2508
L_2	+ ·50	− ·02581	− ·4386 ,,	− ·1538 ,,	− ·1162
L_3	+ ·10	− ·02560	− ·4138 ,,	+ ·1554 ,,	+ ·1061
L_4	−3·0	+ ·00226	+ ·0050 ,,	− ·0069 ,,	− ·0960
L_5	− ·05	+ ·03104	+ ·3357 ,,	− ·1105 ,,	− ·2025
Combined total		− ·00053	+ ·0478 ,,	+ ·0003 ,,	− ·0578

Curvature = + ·010 $\tan^2\phi$.

The field is a little undercorrected, also the distortion, which however refers to a degree of obliquity about ten times as great as the extreme required, so that the distortion at the edge of field will not exceed 1/100 part of that indicated.

This combination was then embodied in actual form, as shown in Fig. 7, having an equivalent focal length of about 3 inches. On adjusting it to give the best performance on a point of light at conjugate focal distances of 7 to 1, the shorter conjugate image being towards the eye, it was found that there was, among other shortcomings, a marked positive zone of aberration. As is well known, a zone of aberration is an effect which follows from balancing off a residuum of spherical aberration of the third or fourth order for the full aperture by an equal residuum of spherical aberration of the second order for the full aperture. In this case the residuum of the third and fourth orders, or at any rate the third order, was negative, and must be reduced or banished.

Therefore the formula for the spherical aberration of the third order was brought into requisition. That formula for a lens having $\mu_D = 1{\cdot}60$ is

$$\frac{y^4}{f^3}\{+\cdot56x^4 - 1{\cdot}058x^3 + 1{\cdot}34\alpha x^3 + 2{\cdot}12x^2 - 2{\cdot}66\alpha x^2 + 1{\cdot}056\alpha^2x^2 - 1{\cdot}203x + 2{\cdot}044\alpha x - 2{\cdot}16\alpha^2x + {\cdot}327\alpha^2x - {\cdot}944\alpha + {\cdot}317\alpha^2 - {\cdot}564\alpha^3 + {\cdot}037\alpha^4 + 1{\cdot}092\}.$$

This formula does not represent all the third order aberration that arises for each lens, but is perhaps the most important one. This being worked out for each of the five lenses gave a sum for the series which was negative in sign and enough in quantity to give rise to the positive zone of aberration previously noticed and also indicated in what direction to redistribute the powers of the lenses so as substantially to reduce this negative sum.

On altering the ratio between F_4 and F_5 from 2:1 to 3:1, while keeping the sum of their powers the same, it was found that the combination then gave a negative zone of aberration, showing as a bright zone outside the focus, implying that the redistribution of powers had gone too far.

After many amendments suggested by the formulae and corresponding trials, the finished lens was completed and got quite free from any visible zone of aberration, yielding a rectilinear and flat anastigmatic field with just enough inward coma to neutralise the slight outward coma of the flat field anastigmatic eyepiece. The focal length of the fourth lens was finally 2·8 times the focal length of the fifth lens (counting the lenses in the order eye to object). Fig. 8 shows a 1 inch micro-objective on the above model together with an anastigmatic flat field eyepiece of

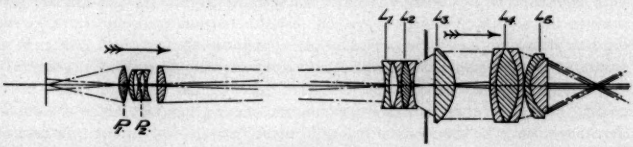

Fig. 8.

·47 inch effective focal length, the diagram being broken in order to save space. With a 7 inch tube the initial magnification is about seven times and that of the eyepiece twenty to twenty-one, thus giving 140 to 147.

As compared with an ordinary microscope with a 1 inch objective we have ten lenses or twenty glass-air surfaces instead of four lenses or eight glass-air surfaces, resulting in a reduction in the light passed to slightly more than one-half, but this alone should not be of practical consequence since for such low powers there is no difficulty in having a superabundance of light. To carry out the same anastigmatic flat field principle in objectives up to N.A. = ·90 to ·95 at least two more simple lenses, the first one being hemi- or hyperspherical, would be necessary, but the construction is quite feasible. Whether it would be commercially worth while to work it out, before the present high costs of production have more nearly approached old levels, is a serious question. On the other hand, the principle could not be carried out except in a half and half sort of way in the case of immersion objectives of large N.A.

So far I have lightly dwelt upon a few out of a much larger number of optical combinations which have been worked out without any resort having been made to ray tracing operations, the system of algebraic formulae as published in my

book, *A System of Applied Optics* (amended edition), having always been found to give quite sufficient guidance. Commander Baker referred at Cambridge to the increasing inaccuracy of the formula $1/(1-x)$ being expanded to one term of the binomial theorem as $1+x$ in cases where x exceeds ·20 or so, and approaches 1 or more, but I cannot say that I have ever been troubled by this difficulty. In cases where it occurs I should not expand $1/(1-x)$ in that way but simply convert $1-x$ into its numerical value and then take the correct reciprocal of it. Let us again consider the case of the special visually corrected triple anastigmat of Fig. 3. How many sets of ray tracings would be required in order to tell us how the lens is going to perform with regard to the four aberrations?

First as to an axial pencil of aperture F/5, we should calculate the ultimate or paraxial foci for the *D* ray by formulae of the first order, then one of two opposite parallel *D* rays entering at the height F/10 in an axial plane would have to be traced, and then if we wanted to know anything about a possible zone of aberration, another *D* ray entering the lens at the height F/14·14 or so would have to be traced through—at least three ray tracings for axial aberration alone, including the tracing of the paraxial ray.

Then taking an F/5 oblique pencil at an angle of say 10°, the penultimate rays close to the oblique principal ray would have to be traced through, two in the primary plane and one in the secondary plane, also two opposite rays each at F/10 distance from the principal ray and lying in the primary plane and also one ray in the secondary plane at F/10 distance from the principal rays would have to be traced. That is another six tracings, making nine tracings so far, if we are to learn anything as to the locus of the ray intersection and the implied flatness of the field and presence of astigmatism and coma as well as the position of the image relative to the axis, on which depends distortion. But a lens of this sort should be capable of a flat anastigmatic field up to 20° from the axis, so another six ray tracings should be carried through for an oblique F/5 pencil entering at 20° angle.

Then again, if it is desired to know whether there is a zone of short readings or long readings in the field, quite a common feature in photographic lenses, it will be necessary to take another set of six ray tracings for a pencil entering at an angle of 14° with the axis, or at any rate this set of tracings may be carried through in place of those carried through for a pencil at the obliquity of 10°. Thus, at the very least fifteen and better still twenty-one ray tracings would be required and even then they would not give us anything like a correct idea of the real nature of the oblique images of points. For the oblique images are not always manifestations of pure spherical aberration, pure astigmatism, or pure coma. The foci are generally more complex and of a nature rendering it difficult to assign the position of best definition or the least circle or patch of confusion. In some photographic lenses, passing a full aperture pencil at a certain degree of obliquity, an image may be found in the form of a bright nucleus and a sort of double coma, i.e. a flare both outwards and inwards, a sort of elongated figure of 8 lying in the primary plane, which is partly due to the state of spherical aberration being quite different in

primary and secondary planes. So that if we wish to get more data on such points we must double or treble the number of our ray tracings. How much time would an expert ray tracer require to carry through and check reliably our minimum of fifteen ray tracings? It would certainly very much exceed the time required for a journeyman optical worker to get out and centre into their mounts a set of lenses according to the skeleton prescription, and for the designer actually to measure off and take notes of the shortcomings of the combination and devise the necessary alterations. And while the journeyman optician is making the trial combination the designer can be spending his time either in thinking out some new design or in devising improvements in some other unfinished combination. All these latter are intelligent operations based upon what the algebraic formulae have to teach, and are absolutely indispensable, whereas the operations involved in ray tracing are purely mechanical, and in themselves teach nothing that cannot be far better learned by the simple inspection of a sample combination. And here I can imagine an ardent advocate of ray tracing calling my attention back again to the subject of microscope objectives of large numerical aperture and asking how I would manage to abolish all need for ray tracing in their case, and he might quote against me the acknowledged policy of the late Professor Abbe and the methods which he advocated. He sought to establish, in place of workshop trial and error methods, more exact forms of mathematical determination which, expressed in cast iron formulae, should enable the necessary data for the lens system to be ascertained and defined once for all, leaving no further occasion or call for improvement. Such a policy would bring the optical designer into line with the bridge engineer. The latter can settle beforehand and on paper every detail of the structure of a great bridge. The formulae relating to the problem have all been worked out with sufficient exactitude to meet the case. It is obvious that this is, by far, a more economical method as to cost than erecting trial bridges, finding out their shortcomings, and then devising improvements to compensate them. The problems before the optical designer are on a vastly smaller scale and of a very different nature, and my contention is that it is more economical as to both time and cost to set a workman to make trial combinations for inspection than it is for the optical designer to spend his much more valuable time in trying to reach the desired end by checking his designs by innumerable ray tracings. Such operations consist of merely mechanical drudgery which should be avoided as much as possible. There is already quite enough drudgery involved in the arithmetical reduction of the algebraic formulae, but these formulae are at any rate as adaptable and intelligent as they are indispensable. The idea of evolving a cut and dried mathematical specification of a new microscope objective, which can be carried out in practice with such confidence that if any samples are found to perform quite wrong then the blame must be attributed to the workman and not to the designer, is certainly a very attractive one which is likely to raise a very strong feeling of astonished admiration in the man in the street, or even in users of microscopes, and we must allow that its adoption has stronger justification in the case of the firm of Carl Zeiss or Leitz than any other firms, for the greater staff costs of designing involved

in the method are more excusable when the results are meant to be turned out by barrels full instead of in the more modest numbers produced by lesser firms.

In the letters of Professor Abbe, published by Professor Cheshire, it will be seen that, in the course of discussing (in 1877) his new projected $\frac{1}{8}$ inch oil immersion objective of N.A. 1·25, he says "The workmanship is exceedingly delicate and difficult especially as to the front (hyperspherical) lens. Among the twenty-four opticians in Mr Zeiss' workshop there is only one who can grind a front lens admitting a balsam angle of 113°, and only one who can make the brass work to it."

Now I admit that this statement by so great an authority seems at first sight to justify his advocacy for a hard and fast mathematical specification of an optical combination before putting it into the workshop, because it is clear that were the first design a merely approximately worked out and tentative one, it would be very difficult to say whether any optical defects that it showed were due to the bad or imperfect design or to mere bad workmanship. But this argument holds good only when we are dealing with a system of very minute lenses, which naturally present very great difficulties in the way of working exactly to specification. I would point out that there is no occasion or necessity whatever for working out a model for a 1/8 or even a 1/12 inch objective on such a small scale as it is finally meant to assume. If I were to attempt to work out such an objective, whether it were an immersion objective of large N.A. or only a dry anastigmat objective of N.A. ·90 or so, I would resort to the obvious expedient of a model on five to ten times the scale of the finished objective and test its performance on conjugate focal distances on the same increased scale. If that were done, the men in the workshops would have a very much more perfect command over the radii and other data and the figuring of the surfaces, so that manifest departures from the ideal optical performance could no longer be attributable to the workshop difficulties. It is well known that the resolving power of microscope objectives depends solely upon their numerical apertures, and that the only objection to using larger focal lengths and larger lenses than what are usual lies in the proportionately increased detriment to definition caused by the larger secondary colour aberrations and residual zones of aberration. So that in the case of large models, although definition of constructional details would be somewhat blurred by secondary waste light, yet there is nothing to prevent the performance of the objective upon an artificial point of light, such as the reflection of a "Pointolite" lamp from a minute globule of mercury, being quite discernable, and just as suggestive of desirable changes for the better to be made in the lens curvatures or other data.

It will be seen that I regard the work of optical designing to be much more analogous to the work of the sculptor than to that of the mechanical engineer. It is well known that the sculptor first makes a rough model in clay embodying the leading idea, and then gradually amends it feature by feature and bit by bit until it approaches the perfected ideal. Or he may make several successive models, each improving upon its predecessor, or presenting varieties in treatment, and not until he has got a satisfactory clay model does he proceed to embody it in marble. In the same way I regard the best optical designing as more of the nature of an

art than the mere calculation and carrying out of a mathematically prescribed specification. The designer must have a good grasp of the algebraic formulae bearing upon his work and of their general tendencies so that he may resort to them for advice and guidance, and he should prefer to study the optical performance of even imperfect trial models rather than start out on the treadmill of ray tracing. The former procedure will sharpen his intelligence and inventiveness as well as his artistic faculties, while the latter practice will tend to atrophy them. The former involves faculties which are quite indispensable while mere ray tracing is generally quite superfluous and unnecessary.

We in this country have a system of optics of our own, founded upon the work of Sir George Airy and Henry Coddington, which only awaits further developments upon useful and practical lines by able mathematicians, if they can only be persuaded to bestow their services and attention to such matters. Above all, the formulae of the third order are in need of further development and completion. Then a great deal of tabulation work might be very useful for reducing the labour in applying some of the more cumbersome algebraic formulae.

I cannot see any strong reasons why we should neglect or discard our British system of geometrical optics and algebraic formulae in favour of any continental system, as so many seem to have been advocating directly, or else indirectly, as, for instance, by not only translating foreign optical works into English, but advocating their adoption as practical text-books.

It seems to me that during the last twenty or thirty years as great a number of new and successful optical combinations have been developed and brought out in this country as in all other countries put together, excepting in the case of microscopic objectives. In conclusion, I venture to think that you will agree that it behoves all of us who constitute this Optical Society to encourage the future development of applied algebraic optics of the British school by all means in our power, rather than slavishly follow the methods of any foreign school, even if our efforts be prompted by no other than patriotic considerations.

DISCUSSION.

Prof. Cheshire: This paper is probably one of the most interesting and valuable pronouncements on the subject of optical designing, for a considerable time. When, some two years ago, we went down to Cambridge for the purpose of trying to interest the Cambridge mathematician in our work, I felt that, since Mr Dennis Taylor was not there to take part in the discussion, we were very much in the position of listening to a performance of the Prince of Denmark, with Hamlet's part cut out. I thereupon took the matter up with Mr Dennis Taylor, and in one letter I pointed out to him that "what especially, I think, the optical world would appreciate from you at the present time is some information on the methods employed by you in the elaboration of such a famous optical system as the Cooke lens. At the present time the optical world seems to be dominated by the broadly-trigonometrical German method. However valuable this method may be in its

detailed application, many of us fail to see that it has any inspiring power. On the other hand, a good many opticians are continually pointing out that the Cooke lens is one that could not have been produced by the German method. Could you not, therefore, sit down and write an interesting paper for us on the genesis and evolution of this lens?" I further impressed upon Mr Taylor that he had a patriotic duty to perform in the matter, and that duty, I think we all agree, has been most generously discharged to-night.

Now first and foremost Mr Taylor stands out as an out-and-out disciple of Coddington, who wrote nearly a hundred years ago, and gave us the fundamental equations upon which the algebraic method of lens designing is based. So much is Mr Taylor satisfied with the sufficiency of Coddington's methods that he tells us that except for a few marginal rays in large telescope object-glasses he has never done a particle of ray tracing. I presume, then, that in his case, the necessary constructional data for the final optical system have, in every case, been approximated to, and worked out, by successive workshop constructions. He agrees that this work could be dispensed with by the use of the trigonometrical method, so that I think the question at issue is clear. In the first place everyone, I take it, with any knowledge of the subject, agrees that the preliminary work in designing new lens systems must be algebraic in character, if it is to be done in the best possible way. The only difference of opinion, then, that can exist is as to whether the final work of getting out the exact constructional data should be done by actual workshop trial and error methods, or drawing office trial and error methods. But we know that the latter method is very largely used on the continent, and also by eminent opticians in this country, so that these facts alone, in my opinion, enable us to answer the question as to what should be taught to the optical student, which is my chief concern in this matter. Clearly, I think, he should be equipped with all the best available tools for his work. He should be taught that both the algebraic and the trigonometrical methods are necessary for his work—one for one purpose and the other for another and quite different purpose. I doubt whether Mr Taylor even would agree that optical students should not be taught the trigonometrical method. I echo, as I am sure we all do, his fervent desire for the establishment of a recognised school of British optics, but I am quite sure that this school cannot be created and maintained in vigorous life, unless our opticians are provided with every available tool for the performance of their work.

Comdr Baker (partly communicated): The paper is of great value on account of the description that it gives of the methods that the author adopts for the purpose of optical design. When it is remembered that the author has been personally responsible for the optical design of all the products of his Firm in addition to the well-known Cooke lens, it is clear that these methods have stood the searching test of practical experience and that they are certainly capable in Mr Taylor's hands of producing satisfactory results.

In broad terms the method of procedure is to design a system initially on Gaussian lines, i.e. the positions, focal lengths, and diameters of the separate lenses are found that will give the required performance. Lenses in this part of the design

are mere abstractions represented by planes perpendicular to the optical axis. First order aberrations are then calculated in accordance with the manner described in the author's *System of Applied Optics*, the formulae of which are based upon earlier work of Coddington and Airy, and the suitable lens forms are evolved that will give the requisite freedom or otherwise from aberrations. This is a procedure that is followed by a number of computing opticians, but Mr Dennis Taylor in the subsequent work pursues a method of his own by having the system made up at this stage in the workshop and examined as regards the residual defects that are due to aberrations of a higher order than the first. The much more customary procedure is to trace trigonometrically a suitable set of rays through the system and determine the magnitude of the residuals from the calculations.

When, therefore, the author states that he hardly ever does any trigonometrical ray tracing it must be remembered that his alternative requires very great skill and experience in order that he may be able to detect from the image defects the nature and magnitude of the residual errors that have to be balanced out by leaving in sufficient aberrations of the first order to counteract them. With his very great knowledge of the appearance of the higher order defects he finds that it is quicker and cheaper to make up the system, corrected for first order, and to let that system do the ray tracing for him. It is very unlikely that any other designer could hope to accomplish the same end until he had equipped himself by years of work with the same mental power of analysis of image defects that Mr Dennis Taylor possesses. Most designers would prefer, I think, to stick to the trigonometrical method, even if it is slower, with the knowledge that it will enable them to make a straightforward analysis of the aberrations that remain.

Mr Dennis Taylor has referred in his paper to the discussion on "The Future of Geometrical Optics" that took place at Cambridge in May, 1921, and to the opening remarks that I contributed to that discussion. I think that he has not entirely appreciated the various points that I wanted to make and I may perhaps be pardoned if I refer to them again. An optical system is designed primarily on Gaussian (i.e. paraxial) lines. The actual rays of light that pass through the instrument will depart both in directions and in positions from the Gaussian rays and it is the designer's business to ensure that these departures shall so balance one another that the emergent light behaves in such a manner that the rays are as nearly as possible in the same positions and directions as rays traced through by paraxial formulae. To do this it is necessary to utilise formulae which give the divergence of the rays after passage and these formulae, calculated first by Coddington, are based upon an expansion in series of the trigonometrical equations for refraction at a surface and of the trigonometrical equations that enable me to pass from one spherical surface to the next. The expansions of these two sets of equations lead to series expressible in ascending powers of the height of intercept upon the stop and the height of intercept upon the lens, or upon the image plane. On account of the axial symmetry of the system these expansions are of an odd degree. The linear terms give the paraxial equations, the cubic terms the aberrations of the first order (spherical aberration, coma, astigmatism, curvature of field, and distortion) and

terms of the fifth and higher orders the residual aberrations. Coddington's formulae are those derived from the cubic terms and if the fields and apertures dealt with are reasonably small the aberrations that they represent are in general all that need be considered, as the others of the fifth and higher powers of the aperture and field are so small as to be negligible. As long, therefore, as we are content with instruments that utilise only small apertures and small fields, Coddington's formulae or others of a similar character are sufficient to enable the designer to work out his instrument. But we have advanced very far since Coddington's time and no one nowadays believes that it is possible to make a workable optical instrument by elimination of first order aberrations alone. Either he sticks to pure trigonometry, and proceeds by trial and error to modify lens shapes until he achieves the best solution he can, or else he utilises algebraic formulae, such as Coddington's, to help him on his way and makes use ultimately of trigonometry to balance the higher order aberrations against first order defects purposely left in. Mr Dennis Taylor's methods are in principle analogous to this second scheme.

It is owing to the enlargement of apertures and fields that have taken place that I have ventured to question whether the method of expression of aberrations by expansion in power series is necessarily the best that can be adopted. Such expansions are very frequently only very slowly convergent, that is to say, aberrations of far higher order than the first must be included before the formula approximates sufficiently closely to the trigonometrical equations from which they are derived. In some cases such expansions are definitely divergent and cannot be used at all.

I should like to note at this point that Mr Dennis Taylor refers to an illustration that I used at Cambridge of what is meant by convergency of a series. The example was intended solely for those members of the audience whose knowledge of mathematics was small. Mr Taylor says that in such cases, where the first terms of the expansion are misleading, he would go back to the equations from which the series are derived. In other words, he must revert to the trigonometrical equations for passage through a surface, or for passage from one surface to the next. Such a statement is surely hardly consistent with his profession that he has never done a trigonometrical ray trace in his life. What I imagine is much more likely is that he calculates the erroneous first order term in this divergent expansion and includes it among the others whose sum total is to be made zero. Subsequently his first order terms are made not zero but something which balances more or less against terms of a higher order and the final balance is the result not purely of calculation but of a combination of calculation with observation of the defects of the trial system. A divergent expansion would in any case be of rare occurrence and it is very unlikely that when it did occur it would have any other effect on Mr Dennis Taylor's method than to make the residuals appear unusually large.

To recapitulate, the aberrations of the first order are insufficient in themselves to enable a full design to be carried out. The expansions of which they form part are too slowly convergent to allow the designer to stop short at the cubic terms. If he may not stop there he must either use the first order terms as a guidance only

and balance them against the higher terms obtained by trigonometry or by observation or he must recast his method entirely. I put aside completely any possibility of calculating aberrations of a higher order in algebraic form; the labour involved is too great.

The whole purport of my remarks at Cambridge was to suggest that the departures, after traversing a lens, of the true ray from the paraxial could possibly be expressed in a form other than that of an expansion and that if a form could be evolved that would permit of successive transfer from one lens to the next and from one space to the next with a final summation of the aberrations at the emergent end of the instrument we might possibly be able to build up an optical calculus that would allow of an instrument being completely computed without having to trace a single ray. To do this satisfactorily we must be prepared to scrap all our present conceptions of the physical aspects of aberrations, because in such a calculus aberrations represent far more than the cubic terms and it is from the cubic terms alone that we derive our ideas of the five aberrations described generally as those of the first order. To take a simple illustration, the first order spherical aberration is a quantity depending upon the cube of the aperture. It leads to a caustic in the form of a semi-cubical parabola. Actually the caustic is probably only a reasonable fit to a semi-cubical parabola in the neighbourhood of the cusp. The optical calculus that I envisage would give a caustic of the same shape near the cusp but departing from the semi-cubical parabola away from the cusp. It would not give the exact shape of the true caustic but only an approximation to the true shape so close that the designer could use it as the true caustic for the whole of the aperture with which he had to deal.

The expression of optical aberrations in a form other than that of a power series opens up endless possibilities. From the thousands of ways in which such aberrations can be expressed a suitable choice can only be made when the following points are observed (i) the function must be closely accurate for all values both of field and of aperture, and (ii) the form must be such as lends itself to successive transference from one medium to the next. I can illustrate the first point by an example taken not from optics but from trigonometry. The value of the tangent of a small angle x (expressed in radians) is also x if the angle is small enough, but as the angle increases a cube term has to be included giving the approximate from $x + \frac{1}{3}x^3$. For greater angles still the term in x^5 must be brought in giving

$$x + \tfrac{1}{3}x^3 + \tfrac{2}{15}x^5.$$

None of those expressions is so near the actual value as

$$\frac{\cdot 873x}{1 - \cdot 390x^2} + \frac{\cdot 127x}{1 + \cdot 058x^2}.$$

For instance when $x = \pi/4$, $\tan x = 1$.

The first term of the series gives ·785, the first two ·914, the first three ·953, whereas the fractional function which utilises only the second power of the variable gives ·961.

The optical formulae that I think ought to be possible are of the same sort of type, but of course nothing like so simple. Where in the illustration the cubic term has a simple numerical coefficient, that would be a function of the first order aberration coefficients, the coefficients, in fact, established by Coddington. The 2/15 of the next term must be replaced by those of the next order aberrations, formulae which, if my memory serves me rightly, occupy nearly a whole page in Mr Dennis Taylor's book.

The more I look at the whole problem the more I am convinced that if we wish to devise a scheme of optical design that will work on some algebraic form dealing with aberrations alone we must get away from the power series expansions and try to devise new forms to serve our needs.

Mr T. Smith: Mr Taylor's paper is of great interest, and we all desire to hear much more from him on this subject in the future. The general scheme of relying on first order aberrations is undoubtedly sound, though I consider that the most advantageous procedure is rather the reverse of that described in the paper. Whether the final adjustment of corrections should be based upon ray tracing or upon an inspection of a trial lens depends upon a variety of circumstances, and one of these is undoubtedly the temperament of the worker. The method laid down by Abbe is perhaps best suited to the German worker, and could be applied anywhere with equal success, but it is not necessarily the best one to adopt in this country among people of a different temperament. Certainly everyone would regret to see the author spending his time in ray tracing or in any routine labour; it would not be making proper use of his ability. The routine work for a highly skilled designer should be carried out by assistants, either in the computing office or in the workshop, the choice depending on a variety of circumstances. Perhaps a wise course is to utilise a combination of the two. The author prefers the workshop alone, and gives among his reasons the amount of ray tracing necessary to obtain the desired information. I cannot help thinking that the author's estimate of the number of rays which should be traced is distinctly high. If rays are traced in the most advantageous manner each can be made to yield information on at least three distinct points. Thus with 15 rays 45 quantities, almost sufficient for the determination of the aberrations of the first four orders, become known. Much of the criticism concerning ray tracing might be valid for the usual trigonometrical methods—on that score doubtless others would speak—but rays can be traced with precision by algebraic methods in which not only are the aberrational terms separated from the others but also a measure is obtained of the first order aberrations and the relative magnitude of those of higher orders is shown. Ray tracing by such a method seems very appropriate for the completion of a design based upon first order expressions. When our ignorance becomes much less extensive than now, no doubt the practice of ray tracing will gradually become obsolete.

Dr L. C. Martin: I think that it would be a very good thing if a flat field high power microscope could be designed for use with an eyepiece micrometer, but possibly the distortion would be too great to allow of the use of such a system for length measurements.

At the Imperial College Professor Conrady has developed a system of lens computing which is not generally known. He first of all uses analytical methods for obtaining a skeleton system, then traces rays in order to get exact numerical values for correcting residual aberrations, and lastly returns to analytical and graphical methods for perfecting the system. This method is, at all events, one of a highly successful nature.

I would like to ask the author whether he can, by examining a model, obtain measurements of aberrations (without much experimental difficulty) which are accurate enough to enable him to make the necessary alterations in the computation of a system.

I agree with Mr Smith that Mr Dennis Taylor rather overestimates the number of ray tracings required. The tracing of a single ray often gives one a considerable amount of information, especially if this ray path is made the basis of certain additional subsidiary calculations.

Mr J. W. Hasselkus (partly communicated): The method of designing optical systems advocated by the author is evidently one in which trigonometrical ray tracing is rigorously excluded. It would appear from the paper that the author uses approximate analytical formulae to arrive at a preliminary design of the optical system under consideration and thereafter makes a series of models beginning with a tentative skeleton of the system right through the scale until the finished article is evolved. The various aberrations are apparently estimated at each step and attempts are made to correct them in the next model and so on until the last model is considered the best performance that can be obtained.

One of the difficulties I see in this method is the uncertainty with which aberrations and their magnitude can be detected and measured. All optical systems are more or less the result of compromises with respect to the aberrations they contain and whatever method of designing is employed, the designer must have the necessary knowledge to distribute the unavoidable aberrations and estimate the value of each as regards its effect in the ultimate image. The trigonometrical method shows all the various aberrations as the work is proceeding and expresses their magnitude exactly, and for this reason alone it would in my opinion lead to definite and reliable results more quickly than a method of designing which depended upon the examination of workshop models, where it would always remain an open question whether a certain defect observed is due to bad design or bad workmanship.

It is surely the higher art in designing to complete your design on paper, whether it be of a bridge or an optical system.

Mr J. Perry: This paper, which may be regarded as reviving the Cambridge discussion upon the future of geometrical, and therewith to a large extent of industrial optics, resolves itself into an advocacy of the use of purely algebraic methods, supplemented only by "workshop trials," and, as might have been expected, justifies its title. It is certainly true that high-class products may be expected from the hands of an artist optician. The question arising, however, is, shall we derive unqualified advantage from the total exclusion of other methods, perhaps more laborious but still more vigorous and precise? Observing that the ultimate end of optical design,

considered as an art and therewith as a practical science, is one, and independent of school, it is of immediate interest to observe the extreme differences of opinion that exist, relative to the means of attainment of that end. As occasionally remarked, the line of cleavage of such differences follows more or less closely along the division of nationality—which suggests much that is supported by reason. When branches of science become closely allied with industry there is a danger of their losing that valuable international character of which on occasion the parental stem is proud to boast. Thus may in part the division of opinion be accounted for.

There is, however, another more fundamental reason that may be assigned for the cleavage. If we but broadly examine the field of science in its historical development we find it pursuing two distinctly different though possibly coterminal courses. Thus, for example, in electromagnetism we see that almost contemporaneously with Faraday who, in his endeavour to form a clear visual impression of electromagnetic phenomena, had enriched the science by the fruitful conception of "lines of force," Neumann was profitably engaged in attacking the problems of the day with the aid of mathematical analysis, thereby leading up to the introduction of the "potential" concept. Standing in similar complementary relationship may be instanced Lord Kelvin and von Helmholtz. Kelvin himself acknowledged that he was never satisfied until he could make a mechanical model of a thing, while, on the other hand, Helmholtz confessed himself happier when in possession of a differential equation. Such instances may easily be multiplied. Even in the formal sciences the same dualism is discernible. Thus, generally speaking, we observe symbolic methods obtaining favour on the continent while unsymbolic methods find here greater support.

These are of course only general tendencies which we are led to consider, and as such are subject to many exceptions. Nevertheless it is readily admissible that even geometrical—and industrial—optics has *not* remained uninfluenced by them. It will, however, be recognised that intronational, and, still more so, international, industrial competition must be founded upon a broader and sounder basis than that offered by racial characteristics. Remembering that the above indicated dualism results from a division, however natural, of Science, and being of the opinion that the best is not to be attained by the slavish following of any particular school, it has seemed to me, in common with many others, that in matters of optical design it is better to steer a course intermediate between the extremes indicated by Mr Dennis Taylor, viz. to use algebraic methods up to the hilt of their applicability (for which certainly some skill and discretion is requisite), supplementing them, however, by rigorous calculation, the labour and time necessarily expended upon which is, I think, somewhat overestimated by Mr Dennis Taylor.

Dr J. S. Anderson: The question raised by Dr Martin as to the accuracy of aberration measurements is an important one. There is certainly room for improvement in the methods of measuring aberrations in optical systems, and I think that real progress can only be made by the systematic study of the amounts of the different aberrations that can be tolerated. It is to be hoped that considerable progress in this direction will be made in the next few years.

Mr Dennis Taylor (partly communicated): First, in reply to Professor Cheshire's remarks, I may say while I agree that the processes of trigonometrical ray tracing, and especially the tracing of rays of pencils converging to, or diverging from, points on the axis and passing through the edge and intermediate zones of objectives that are of large absolute as well as relative aperture*, and yet have to be very perfectly corrected against axial spherical aberration, should be taught to the student, yet I think he should be made to grasp that it is chiefly to the general algebraic system that he must trust for his designing, and he should be encouraged to familiarise himself with the various implications of the formulae under the many different conditions that may arise.

Personally, however, I fear that I should make a very bad teacher, there is so much that is very difficult to formulate and impart. Of the ray tracing methods I have laid more stress on the desirability of dealing with rays of axial pencils through large aperture objectives, first, because it presents a simpler case than tracing rays of oblique pencils, and also because more or less perfect freedom from spherical aberration or zonal aberration at the foci of axial pencils is so frequently required. The operation in such cases is much more profitable than in so many other cases, wherein I still maintain that the most sure and expeditious way is to study the performance of tentative models by actual inspection. As regards the usual class of achromatic objectives for telescopes, whose apertures exceed one-fifteenth part of their focal lengths, it may be truly said that the algebraic formulae for freedom from spherical aberration and coma are quite sufficient and exact enough in themselves without troubling to resort to any ray tracing whatever.

Two other critics consider that I have exaggerated the number of ray tracings which should be required in order to replace actual trials, and perhaps with regard to some cases I may have done so, but I have come across cases of very oblique images formed by photographic and other lenses of large field whose real character and mean general effect would require far more ray tracings to elucidate than any computer should be justified in working out.

Mr T. Smith has observed that "if rays are traced in the most advantageous manner, each can be made to yield information on at least three distinct points." Mr Smith with his high mathematical endowments is able to deal with such problems from a transcendental standpoint which enables him to draw deductions which I am afraid would entirely escape most of us, and I know that although, like the arrow in the song, I cannot follow him in his flight, yet he has something real in his mind which is beyond my grasp. I have been unable to follow any of his formulae, partly because I cannot find any statement of what his symbols stand for. I am afraid that most writers of papers involving mathematical formulae, including myself, so often fail to define the meaning of their symbols with sufficient clearness, and that fact, added to the inherent difficulty experienced by many of us, and especially myself, in following the lines of mathematical reasoning adopted by other writers,

* It is obvious that a trial model of an objective of under 2 or 3 inches aperture can be made easily and cheaply while there are practical objections of very much greater cost, etc., which render the use of trial models of much larger apertures undesirable.

generally makes us far less appreciative than we should be of other men's work among these abstruse optical problems.

Both Commander Baker and Mr Smith tell us they have been busy for years in trying to develop formulae of greater exactitude, and the latter tells us that "rays can be traced with precision by algebraic methods in which not only are the aberrational terms separated from the others but also a measure is obtained of the first order aberrations and the relative magnitude of those of higher orders is shown." This is interesting and full of promise, and I may say that I heartily wish that practical success will ultimately attend their arduous undertakings.

In the meantime, I think I may claim to have proved that in the actual practice of optical designing my system of second order formulae founded upon Airy and Coddington furnishes a quite reliable guide and inspiration sufficient for the great generality of cases that arise. I have very seldom found the aberration of higher orders as large, or nearly as large, as those of the second order; in fact, I can only recall one case in which the aberrations of the third and higher orders were so strong as more or less to submerge the aberrations of the second order. This was a case of an anastigmatic flat field erecting eyepiece which I had had in my mind off and on for many years. I worked out a design for one which held out a fair promise of success according to the second order formulae, but it involved, in order to fulfil the Petzval condition, the introduction of some unusually deep curved lenses, so that I was quite prepared to find, as I did find on trial of the model, that after ten degrees from the centre of the apparent field had been passed the aberrations of higher orders began to assert themselves so strongly as to render the combination quite useless. Not only do aberrations of the higher orders tend to assert themselves in the case of deeply curved and powerful lenses used at large relative apertures, but also in cases where lenses are not necessarily powerful in themselves, but are nevertheless deeply curved owing to their being strongly convex on one side and concave on the other, or of meniscus shape. This is especially the case when, in addition, the lens is so presented to the pencil of rays that the edge rays are refracted through the lens edge at violent obliquity so that of the two refractions taking place one deviates the ray, say, downwards and the other upwards in different degrees.

This corresponds to the case of a simple prism refracting a ray which enters at a very large angle of incidence. The ray is deviated in opposite directions of rotation at the two refractions, and the prism is then so set relatively to the ray as to be very far removed from the condition of minimum deviation, when extra refraction, expressed as higher developments of the radian measure of the swing from the position of minimum deviation, gradually comes into action. For instance, an achromatic rangefinder prism, giving $16'\ 51''$ of minimum deviation, swinging round on an axis parallel to its refracting edge through an angle θ gives an excess deviation which is strictly proportional to θ^2 until $15°$ of swing is reached after which a noticeable extra deviation proportional to higher powers of θ sets in, amounting at $\theta = 42°$ to $152''$ more than the above formula gives. Here θ is the simple radian measure of the angle of swing. The obvious corollary to this is that the spherical aberration incurred by rays of axial pencils traversing the edges of lenses is in many

cases largely due to the unequal refractions taking place at the two surfaces (i.e. departures from minimum deviations), as well as the approach or retreat of the lens edges to or from the radiant point and the Gaussian focal point. The corrections for the latter displacements are matters of the vergency of the rays and of the versines corresponding to the two curvatures, expressed by the parameters α and x respectively. This aspect of the case suggests thoughts worth further pursuing*. I might here mention that a certain enquiry into the case of the swinging prism, undertaken a few years ago by Mr Ralph Cheshire, led him to the conclusion that the higher functions of θ required to express the higher orders of excess refraction formed a series too slightly convergent to be of much use; but possibly there are other ways of attacking the problem by developments of other trigonometrical functions of θ.

In conclusion I feel I must thank you for the kindly spirit in which you have received and discussed my somewhat unorthodox paper. I, for my part, have felt myself most amply repaid by the highly interesting discussion which has followed it.

* Using Coddington's spherical aberration formula and notation we have as the condition that the angles of entrance and emergence at the two surfaces are equal (the condition of minimum prismatic deviation)

$$\frac{1}{r}+\frac{1}{u}=\frac{1}{s}+\frac{1}{v} \text{ or } \frac{1+x}{2(\mu-1)f}+\frac{1+\alpha}{2f}=\frac{1-x}{2(\mu-1)f}+\frac{1-\alpha}{2f},$$

in which r = 1st radius, s = 2nd radius, u = distance from lens to point of divergence or convergence of the entering pencil, and v = distance from lens to point of convergence or divergence of the emergent pencil, and from this equation we get $x=-(\mu-1)\alpha$. If this value is inserted in place of x in the aberration expression

$$\frac{y^3}{8f^3\mu(\mu-1)}\left\{\frac{\mu+2}{\mu-1}x^3+4(\mu+1)\alpha x+(3\mu+2)(\mu-1)\alpha^2+\frac{\mu^3}{\mu-1}\right\},$$

all the first three terms are eliminated and the expression becomes

$$\frac{y^3}{8f^3\mu(\mu-1)}\left\{\frac{\mu^3}{\mu-1}\right\} \text{ or } \frac{y^2}{8f^3}\left(\frac{\mu^2}{(\mu-1)^2}\right).$$

THE DISTRIBUTION OF CORRECTIVE DUTIES IN OPTICAL INSTRUMENTS

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MS. received, 16th February, 1923. *Read and discussed, 8th March*, 1923.

ABSTRACT. The conditions which should be satisfied by the components of a complex system are determined in a convenient form in terms of the aberrational conditions which the complete instrument has to satisfy. The use of the method evolved is illustrated by a lens of three elements such as the "Cooke" photographic lens.

The main problem with which the optical designer has to deal is the removal or reduction of a number of aberrations in an optical system which satisfies certain requirements concerning dimensions, positions of lenses, field of view, and so on. In some cases the attainment of specified aberrations and not their removal is the aim. Systems otherwise differing widely may be equally satisfactory in their corrections, and other conditions, such as comparative costs of production, then determine which shall be adopted. From the scientific point of view these latter considerations are accidents and may be neglected, and the removal of aberrations retains its place as the crucial factor in design. Especially is this the case when the conditions which the ideal instrument should satisfy cannot be separated into two groups, the one containing conditions of predominant, the other of secondary, importance. Where such a division is possible the scientific principles of design seem to be well understood, but in the general and more important case, well illustrated by photographic lenses, the adequacy of existing theory is less clear. There appears to be no recognised method of investigation which ensures that every system considered in detail will automatically fulfil the conditions initially laid down for observance. The present paper deals with the foundations of a theory designed to remove this deficiency. By following the procedure here outlined only systems which fulfil the conditions are obtained. Whether the systems can actually be constructed depends on a number of considerations outside the scope of this enquiry—for example the existence of suitable kinds of glass. If the solution obtained is unsatisfactory in any of these latter respects it is necessary to make a change in one or more of the initial assumptions and repeat the calculations on this modified basis. In this way, either a satisfactory solution will be obtained or alternatively the conclusion is drawn that no acceptable solution exists.

It is generally admitted that in nearly all optical systems aberrations not of the lowest order must be taken into account, since systems corrected by considering the terms of the lowest order alone may perform indifferently or worse. Nevertheless it is correct to base the design of instruments on aberrations of the lowest order, for it is quite certain that no instrument with large aberrations of this kind can prove satisfactory. In general instruments of good performance will not be mathematically corrected for aberrations of the lowest order, but the amount of these aberrations allowed to remain will be small, the magnitudes and signs being chosen to diminish as far as possible the evil effect of higher order errors which the designer has not found it possible to eliminate. These considerations lead to a routine in which the work of designing an instrument is divisible into three processes:

(1) the removal of the lowest order aberrations on the assumption that higher order aberrations are zero;

(2) the evaluation of the higher order aberrations in the system derived from the previous process;

(3) the introduction of small amounts of the lower order aberrations to compensate for the defects found from the second process.

The procedure in the first and third operations is the same, but the conditions imposed are somewhat changed. Processes (2) and (3) may have to be repeated once or more before the desired conditions are observed to the highest accuracy attainable. In most cases it will be found that processes (1) and (3) may be applied to a simplified system of lenses for which the calculations are shorter instead of to the real system it is proposed to construct, but this does not apply to the calculations under (2) from which the whole aberrations present are found. It is however possible to construct a plan of operations in which the calculations under (2) relate to the simplified system also, corrections on a known basis being applied to this system to yield the real system.

In the construction of an optical instrument five different degrees of freedom may be distinguished. These relate to

(1) the focal lengths of the component lenses,

(2) the separations of the component lenses,

(3) the shapes of the component lenses,

(4) the refractive indices of the glasses*,

(5) the dispersions of the glasses.

Of these (1) and (2) alone affect the general character of the final instrument, and variations in their values for the purpose of correcting aberrations will be subject to strict limitations. As a rule the thicknesses of lenses may be neglected, the primary effects of the thickness being taken into account by measuring separations between the principal points of the component lenses. For the removal of spherical aberrations variations of shape and of refractive index may be used freely. The accepted method of design at the present time involves a choice of index (in the light of past experience together with further limitations dependent on the kinds

* The use of aspherical surfaces produces effects similar to changes of refractive index.

of glass in stock or certainly obtainable from the glass maker) as a preliminary step, thus leaving the shape of the components as the only means of satisfying the spherical conditions. This procedure is objectionable on several grounds. In the first place it is not possible by changes of shape to secure all the effects attainable by variation in the refractive index. Further no indication is obtained whether distinct advances would ensue from the production of new glass having certain novel combinations of properties, so that the glass maker does not receive from the optician the guidance he has a right to expect; and, contradictory as it may appear, this procedure leads to a demand for a greater number of types of optical glass than are actually needed. The alternative method to be suggested retains for the correction of the spherical aberration the full degrees of freedom represented both by (3) and (4), and in fact the choice of refractive indices becomes the last, and not the first, step in the process of removing the spherical errors.

The dispersions of the glasses must be chosen to satisfy the conditions for the removal of chromatic aberrations. The existing methods are quite satisfactory for this purpose and will not be discussed here.

Before proceeding to the consideration of the general case, attention may be called to the existence of special solutions which are based upon a definite method of correction. It will suffice to give a single example, in which the image of an object at infinity is to be freed from all aberrations, utilising achromatic objectives, which may be regarded as 'thin,' each producing an image free from spherical aberration and coma. The image produced by the complete lens is evidently free from these two aberrations and from colour, but the remaining spherical errors must be eliminated by selecting suitable focal lengths and separations for the components since the degrees of freedom specified under (3), (4) and (5), apart from a small degree of freedom as regards (4), have been absorbed already. Three conditions remain to be met. Of these the Petzval condition involves the refractive indices, leaving two conditions to be satisfied by the selection of the powers and positions of the lenses. The complete specification of the system, consisting say of three separated lenses, can therefore be expressed in terms of three variables, of which one, here represented by l, is merely a unit of length. The conditions are satisfied by making

$$\left.\begin{aligned} l\kappa_1 &= u - 1 + (u - v)^{-1} \\ l\kappa_2 &= 1 - u \\ l\kappa_3 &= (v - u)^{-1} \\ t_1\kappa_1 u &= t_2\kappa_3 v = 1 \end{aligned}\right\} \qquad \ldots.(1),$$

so that

$$\kappa_1 + \kappa_2 + \kappa_3 = 0 \qquad \ldots.(2),$$

and

$$\left.\begin{aligned} l\kappa_{1,3} &= 1 - u^{-1} - v^{-1} \\ \frac{\partial\kappa_{1,3}}{\partial\kappa_3} &= uv^{-1} \\ l\frac{\partial^2\kappa_{1,3}}{\partial\kappa_1\partial\kappa_3} &= -t_1t_2u^2 \end{aligned}\right\} \qquad \ldots.(3).$$

Since the curvature differences between each successive object and image are directly additive it is evident that (2) and the Petzval condition

$$\kappa_1 \varpi_1 + \kappa_2 \varpi_2 + \kappa_3 \varpi_3 = 0 \qquad \ldots.(4)$$

together ensure the correction of the curvature and astigmatism. The condition (4) is most simply met by using the same glasses for each achromatic lens, the relation (2) then involving (4) also. The distortion is corrected if

$$\Sigma \kappa_m \frac{\partial \kappa_{m,n}}{\partial \kappa_m} \Big/ \frac{\partial \kappa_{1,m}}{\partial \kappa_m} = 0,$$

and since

$$\frac{\partial \kappa_{1,1}}{\partial \kappa_1} = 1, \qquad l\kappa_1 \frac{\partial \kappa_{1,2}}{\partial \kappa_1} = u - u^{-1} - uv^{-1} + (u - v)^{-1},$$

$$\frac{\partial \kappa_{1,2}}{\partial \kappa_2} = 1 - u^{-1}, \quad l\kappa_2 \frac{\partial \kappa_{2,3}}{\partial \kappa_2} = (1 - u)(1 - v^{-1}),$$

$$\frac{\partial \kappa_{1,3}}{\partial \kappa_3} = uv^{-1}, \qquad l\kappa_3 \frac{\partial \kappa_{3,3}}{\partial \kappa_3} = (v - u)^{-1},$$

this is identically satisfied. If in addition it is required to secure freedom from curvature, astigmatism, and distortion at all magnifications, the stop must be effectively placed in one of the unit planes, and u and v, instead of being independent, will be connected by the relation

$$(5 + 2\varpi)(uv - u - v) = (2u - 1)(u - 1) \qquad \ldots.(5),$$

where ϖ is the Petzval coefficient for a component lens. In any practical application it is probable that $\frac{\partial \kappa_{1,3}}{\partial \kappa_3}$ would be specified, thus fixing the working distance in terms of the focal length. This gives the ratio of u to v, and from (5) two values are obtained for u, of which one is usually positive and one negative. A suitable value for l is obtained from the first of equations (3), and equations (1) then enable the entire system to be specified.

In the more important general case the procedure is rather the reverse of that followed above, inasmuch as the positions and powers of the lenses and the positions of the stop and image are given, and the method of correcting the individual lenses has to be found. Consider a system of n lenses of which a typical member is distinguished by the letter m. Let the magnifications for the object and for the stop in the entire instrument be G and S respectively, and the corresponding quantities for the particular component g_m and s_m, so that

$$G = g_1 g_2 \ldots g_n, \quad S = s_1 s_2 \ldots s_n.$$

Moreover suppose that the images formed by the mth lens are magnified G_m and S_m times when converted into the final object and stop images, so that

$$G_m = g_{m+1} g_{m+2} \ldots g_n, \quad S_m = s_{m+1} s_{m+2} \ldots s_n.$$

The law of addition for first order aberrations is then

$$\epsilon_v = \Sigma \epsilon_{v,m} G_m^v S_m^{4-v} \quad (v = 0, 1, 2, 3, 4) \qquad \ldots.(6),$$

where ϵ_v is the linear coefficient for the aberration of the complete lens and $\epsilon_{v,m}$ is the corresponding coefficient for the component m, the summation extending over the whole of the components of which the system is constructed. The aberrations corresponding to different values of v are as follow:

$v = 0$ spherical aberration of stop centre,
$v = 1$ distortion,
$v = 2$ curvature of primary focal surface,
$v = 3$ coma,
$v = 4$ spherical aberration.

The curvature of the secondary focal surface is derived from the curvature of the primary surface in conjunction with the Petzval coefficient ϖ which is calculated from the formula

$$\kappa\varpi = \Sigma\kappa_m\varpi_m \qquad \ldots.(7),$$

κ being the power for the complete lens and κ_m that of the typical component. The simple form of this relation renders the method of obtaining a definite value of ϖ for the complete lens sufficiently obvious, and this sum may therefore be left out of account in the general investigation, which will accordingly concern the aberrations of series zero alone.

It may be observed that equation (6) is true whether ϵ_0 and $\epsilon_{0,m}$ relate to the stop aberration strictly or to the complete coefficient which occurs in potential functions*. The difference between the two in the complete system is a numerical multiple of $(S - G)(S^3 - G)/G\kappa$, and in the component is the same multiple of $(s_m - g_m)(s_m^3 - g_m)/g_m\kappa_m$. These terms make a contribution to the sum on the right of equation (6) equal to

$$\Sigma (s_m - g_m)(s_m^3 - g_m) S_m^4/g_m\kappa_m,$$

or, since

$$(s_m - g_m) S_m G_m/\kappa_m = (S - G)/\kappa,$$

the sum becomes

$$\frac{S - G}{\kappa} \Sigma \left(\frac{S_{m-1}^3}{G_{m-1}} - \frac{S_m^3}{G_m}\right)$$

or

$$\frac{S - G}{\kappa}\left(\frac{S^3}{G} - 1\right),$$

which is the corresponding term for the complete lens.

Let the aberration coefficients for the component m now be referred to arbitrarily selected object and stop planes for which the magnifications with respect to this lens are D and T respectively. Denote these new aberration coefficients by

$$(T - D)^4\eta_v \qquad (v = 0, 1, 2, 3, 4).$$

Then if the complete coefficients are taken†,

$$\epsilon_{v,m} = (\eta_0, \eta_1, \eta_2, \eta_3, \eta_4 \mathfrak{Y} s_m - D, - s_m + T)^{4-v}(g_m - D, - g_m + T)^v + (g_m - D)(s_m^{4-v} g_m^{v-1} D^{-1} - 1)/\kappa_m \qquad \ldots.(8),$$

* *Trans. Opt. Soc.* 23 (1921–22) 315. † *Loc. cit.* p. 319.

and the formula for ϵ_v becomes

$$\epsilon_v = \Sigma\,(\eta_0, \eta_1, \eta_2, \eta_3, \eta_4 \mathbin{\text{⨂}} S)^{4-v}_m\,(G)^v_m + \Sigma \frac{g_m - D}{\kappa_m}\left(\frac{S^{4-v}_{m-1}\,G^v_{m-1}}{g_m D} - S^{4-v}_m\,G^v_m\right) \qquad \ldots.(9),$$

where $(S)_m$ is written as an abbreviation for

$$(S_{m-1} - S_m D, -S_{m-1} + S_m T).$$

Consider the more general function E' defined by

$$E' = \Sigma\,(\eta_0, \eta_1, \eta_2, \eta_3, \eta_4 \mathbin{\text{⨂}} \lambda)_m\,(\mu)_m\,(\nu)_m\,(\xi)_m + \Sigma \frac{g_m - D}{\kappa_m}\left(\frac{\lambda_{m-1}\mu_{m-1}\nu_{m-1}\xi_{m-1}}{g_m D} - \lambda_m\mu_m\nu_m\xi_m\right) \qquad \ldots.(10),$$

where the λ's etc., like the S's and G's, form a consistent chain of magnifications running through the system, the four object planes corresponding to the four letters λ, μ, ν, ξ being chosen arbitrarily. Since two paraxial rays meeting the axis in different points determine completely all paraxial magnifications, the λ's etc. may be expressed in terms of the S's and G's in the form

$$\lambda_m = a S_m + b G_m,$$

where a and b are independent of m. Since λ, S, and G all become unity when n is written for m, and the suffix o is equivalent to the absence of a suffix, the relation becomes

$$(S - G)\,\lambda_m = (\lambda - G)\,S_m + (S - \lambda)\,G_m \qquad \ldots.(11).$$

Substitute this value in the expression for $(\lambda)_m$ and similarly with $(\mu)_m$, $(\nu)_m$ and $(\xi)_m$. Equations (9) and (10) then give*

$$E'\,(S - G)^4 = (\epsilon_0', \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4 \mathbin{\text{⨂}} \lambda)\,(\mu)\,(\nu)\,(\xi) \qquad \ldots.(12),$$

where $$(\lambda) = (\lambda - G, -\lambda + S),$$

and similarly for (μ), (ν), (ξ). In other words, however λ, μ, ν, ξ are chosen, the value of E' in equation (10) may be determined from the initial aberrational requirements. Values may now be assigned to λ, μ, ν, ξ which cause the sums on the right of (10) to assume very simple forms. In particular values may be chosen which cause a number of the η's to disappear from the equation through their coefficients becoming zero. Further the T and D for each lens may be selected so that some of the η's have values independent of the shape of the lens and of the materials of which it is made, while others may involve one but not both of these variables. A suitable selection of the quantities available thus enables particular variables to be given proper values, one by one, which in combination in the complete lens automatically involve the satisfaction of the required aberrational conditions.

The method of eliminating particular coefficients will be evident from (10). Thus suppose η_4 is to be eliminated for the component m, it is only necessary to choose λ so that

$$\lambda_{m-1} = \lambda_m T,$$

* The accents are introduced as a reminder that non-aberrational magnitudes are included.

or in other words the plane corresponding to λ is an image of the stop plane associated with this component. To eliminate both η_3 and η_4 one of the other planes μ, ν, ξ is identified with this λ plane. The coefficients η_3 and η_4 for lens m, η_0 for lens p, and η_4 for lens q may be removed by the choice

$$0 = \lambda_{m-1} - \lambda_m T_m = \mu_{m-1} - \mu_m T_m = \nu_{p-1} - \nu_p D_p = \xi_{q-1} - \xi_q T_q,$$

and so on.

There is one particular choice for D and T which offers such special advantages that it will almost invariably be convenient to adopt it. These arise from the circumstance that thickness of glass is rarely introduced into lenses as an independent variable influencing aberrations, but assumes a value dependent on the aperture of the lens. Accordingly it is permissible when first order aberrations alone are considered to neglect these thicknesses, the real lenses thus being replaced by 'thin' lenses. Assuming then that the component lenses are thin, put $T = 1$, $D = -1$. Then

$$\left.\begin{aligned} 4\eta_0\kappa &= 1 \\ \eta_1 &= 0 \\ 4\eta_2\kappa &= 1 + \varpi/3 \\ 4\eta_3\kappa &= -\beta \\ 4\eta_4\kappa &= \gamma \end{aligned}\right\} \qquad \ldots.(13)$$

in the notation previously employed for thin lenses*, so that apart from proportionality to the focal length of the lens η_0, η_1 and η_2 are practically constants, η_3 depends on the shape of the unit surfaces, that is practically the shape of the lens, and when this is determined η_4 depends upon the glasses of which the component is made.

It will be noted that the terms in (10) involving the η's are independent of both S and G, so that if these are changed in value most of the numerical work is unaffected. Also if the aberrational conditions are changed, as for instance in correcting the system after the higher order aberrations have been evaluated, it is only necessary to insert revised values of the E''s calculated from (12) in the old equations.

The only circumstance when equation (10) is not in a suitable form for direct calculation is when one or more of the terms in the second sum become infinite through the vanishing of one of the g's. In this case the sum may be written

$$\Sigma \frac{h_m - D}{\kappa_m}\left(\frac{\lambda_{m-1}\mu_{m-1}\nu_{m-1}\xi_{m-1}}{h_m D} - \lambda_m\mu_m\nu_m\xi_m\right)$$
$$+ \Sigma \frac{g_m - h_m}{\kappa_m}\left(\frac{\lambda_{m-1}\mu_{m-1}\nu_{m-1}\xi_{m-1}}{g_m h_m} - \lambda_m\mu_m\nu_m\xi_m\right),$$

where the h's correspond to a magnification for which the terms of the first line are always finite. The second sum simplifies at once to

$$\frac{G-H}{\kappa}\Sigma\left(\frac{\lambda_{m-1}\mu_{m-1}\nu_{m-1}\xi_{m-1}}{G_{m-1}H_{m-1}} - \frac{\lambda_m\mu_m\nu_m\xi_m}{G_m H_m}\right),$$

that is to

$$\frac{G-H}{\kappa}\left(\frac{\lambda\mu\nu\xi}{GH} - 1\right).$$

* See for instance *Trans. Opt. Soc.* 22 (1920–21) 112.

Now from (12) the term in E' which does not represent an aberration is

$$-\frac{S^3-G}{(S-G)^3\,G\kappa}(\lambda-G)(\mu-G)(\nu-G)(\xi-G).$$

Thus if E is defined by

$$E(S-G)^4=(\epsilon_0,\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4\,\underset{\sim}{\lambda})(\mu)(\nu)(\xi) \qquad \ldots.(14),$$

where every coefficient is now a strict measure of an aberration, the general equation for the calculation of the system becomes

$$\begin{aligned}&\Sigma(\eta_0,\eta_1,\eta_2,\eta_3,\eta_4\,\underset{\sim}{\lambda})_m(\mu)_m(\nu)_m(\xi)_m\\&\quad+\Sigma\frac{h_m-D}{\kappa_m}\left(\frac{\lambda_{m-1}\mu_{m-1}\nu_{m-1}\xi_{m-1}}{h_mD}-\lambda_m\mu_m\nu_m\xi_m\right)\\&\quad=E-\frac{G-H}{\kappa}\left(\frac{\lambda\mu\nu\xi}{GH}-1\right)\\&\qquad-\frac{S^3-G}{(S-G)^3G\kappa}(\lambda-G)(\mu-G)(\nu-G)(\xi-G)\quad\ldots.(15).\end{aligned}$$

When $G=0$ the second and third terms on the right when combined become

$$-\frac{1}{\kappa}(1/H+3/S-1/S^3,-1,0,0,H\,\underset{\sim}{\lambda},1)(\mu,1)(\nu,1)(\xi,1),$$

the value of which is always finite. Since H is arbitrary its value may be selected to simplify the calculations in any desired way. One possible choice is to identify it with S, but a better appears to be to cause it to coincide with λ, μ, ν or ξ. Taking $H=\xi$ by way of example the sum on the left of (15) becomes

$$\Sigma\frac{\xi_{m-1}-\xi_mD}{\kappa_m}\left(\frac{\lambda_{m-1}\mu_{m-1}\nu_{m-1}}{D}-\lambda_m\mu_m\nu_m\right).$$

If a more symmetrical expression is desired H may be given in turn the values λ, μ, ν, ξ and the mean taken. The equation then takes the form

$$\begin{aligned}&\Sigma(\eta_0,\eta_1,\eta_2,\eta_3,\eta_4\,\underset{\sim}{\lambda})_m(\mu)_m(\nu)_m(\xi)_m\\&\quad+\Sigma\frac{1}{\kappa_m}(1/D,-\tfrac14,0,-\tfrac14,D\,\underset{\sim}{\lambda}_{m-1},\lambda_m)(\mu_{m-1},\mu_m)(\nu_{m-1},\nu_m)(\xi_{m-1},\xi_m)\\&\quad=E-\frac{S^3-G}{(S-G)^3\,G\kappa}(\lambda-G)(\mu-G)(\nu-G)(\xi-G)\\&\qquad+\frac{1}{\kappa}(1/G,-\tfrac14,0,-\tfrac14,G)(\lambda,1)(\mu,1)(\nu,1)(\xi,1)\qquad\ldots.(16).\end{aligned}$$

Now insert for T and D their special values 1 and -1 respectively. The equation for the calculation is then most conveniently arranged in the form

$$\begin{aligned}&\frac{\kappa}{4}\Sigma\frac{1}{\kappa_m}(0,0,\tfrac12+\varpi/3,-\beta,\gamma\,\underset{\sim}{\lambda})_m(\mu)_m(\nu)_m(\xi)_m\\&\quad=(S-G)^{-4}(\epsilon_0,\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4\,\underset{\sim}{\lambda})(\mu)(\nu)(\xi)\\&\quad-\frac{S^3-G}{(S-G)^3G}(\lambda-G)(\mu-G)(\nu-G)(\xi-G)+\frac{\lambda\mu\nu\xi}{G}+G\\&\quad+\tfrac18(\lambda-1)(\mu-1)(\nu-1)(\xi-1)-\tfrac18(\lambda+1)(\mu+1)(\nu+1)(\xi+1)\quad\ldots.(17),\end{aligned}$$

where the e's are the aberration coefficients desired for the complete lens in their non-dimensional form. The absence on the left of the first two terms should be particularly noted. This circumstance makes this the most convenient equation to use in actual calculations. It will be observed also that at least two of the factors $(\lambda)_m$, $(\mu)_m$, $(\nu)_m$, $(\xi)_m$ will be effectually divisible by κ_m when these quantities are calculated by the usual paraxial relations, and that therefore in all calculations κ_m will occur as a multiplier and not a divisor. If the second member of one of the factors, say $(\xi)_m$, has been made zero, the first member is $2\Big/\frac{\partial\kappa_{m,n}}{\partial\kappa_m}$, or, what is the same thing, $2\xi\Big/\frac{\partial\kappa_{1,m}}{\partial\kappa_m}$. The value of ξ_p is given by

$$\frac{\partial\kappa_{m,n}}{\partial\kappa_m}\xi_p = \frac{\partial\kappa_{m,p}}{\partial\kappa_m} \quad \text{or} \quad \frac{\partial\kappa_{p+1,m}}{\partial\kappa_m},$$

so that

$$\frac{\partial\kappa_{m,n}}{\partial\kappa_m}(\xi_p - \xi_{p-1}) = \kappa_p \frac{\partial^2\kappa_{m,p}}{\partial\kappa_m\partial\kappa_p} \quad \text{or} \quad -\kappa_p\frac{\partial^2\kappa_{p,m}}{\partial\kappa_p\partial\kappa_m},$$

according as m is less or greater than p. It at once follows that if λ, μ, ν, ξ have been chosen to eliminate β_p, γ_p, β_m, γ_m from the relation the contribution to the sum from these two components is

$$\kappa\left(\frac{\dfrac{\partial^2\kappa_{m,p}}{\partial\kappa_m\partial\kappa_p}}{\dfrac{\partial\kappa_{m,n}}{\partial\kappa_m}\dfrac{\partial\kappa_{p,n}}{\partial\kappa_p}}\right)^2\left(\frac{\kappa_m+\kappa_p}{2}+\frac{\kappa_m\varpi_m+\kappa_p\varpi_p}{3}\right),$$

which indicates that the simple sum of the powers of the lenses, like the Petzval sum, is an important factor in the removal of aberrations. It will be recalled that in the example given earlier of special solutions of aberrational problems it was found necessary to make both these sums equal to zero.

Before leaving this aspect of the problem one more formula may be noted. If the factors containing λ, μ, ν, ξ are arranged as homogeneous products of the fourth degree in $\lambda - G$, $\lambda - S$, and so on, equation (16) becomes

$$\begin{aligned}
&\Sigma\,(\eta_0, \eta_1, \eta_2, \eta_3, \eta_4\overset{\circ}{\lambda})_m\,(\mu)_m\,(\nu)_m\,(\xi)_m \\
&\quad -\tfrac{1}{4}\Sigma\frac{1}{(T-D)^3\kappa_m}(0,\ 3-3T^2,\ 2-2TD,\ 1-D^2,\ 0\overset{\circ}{\lambda})_m\,(\mu)_m\,(\nu)_m\,(\xi)_m \\
&= (\eta_0, \eta_1, \eta_2, \eta_3, \eta_4\overset{\circ}{\lambda})\,(\mu)\,(\nu)\,(\xi) \\
&\quad -\frac{1}{4\,(S-G)^3\,\kappa}(0,\ 3-3S^2,\ 2-2SG,\ 1-G^2,\ 0\overset{\circ}{\lambda})\,(\mu)\,(\nu)\,(\xi)\ \ldots.(18)^*,
\end{aligned}$$

* The method of this paper applied to equation (21) of vol. 23, p. 319, gives the result that $(S-G)^{4w-2n}(D^w_{2w}, D^w_{2w+1}, \ldots D^w_{2n-2w})(P_{2w}-G, S-P_{2w})(P_{2w+1}-G, S-P_{2w+1})\ldots(P_{2n-2w}-G, S-P_{2n-2w})$ is independent of S and G whatever values may be assigned to the P's. Equation (21) is a special case of this result. Equation (23) with $w=0$ is brought within the scope of this relation by replacing the D's by Δ's where

$$\Delta^0_w = D^0_w + \frac{2n-v}{2n}(S-G)(S^{2n-v-1}G^{v-1}-1),$$

equation (18) above involving the special case $n=2$. [Correction: In equation (26) of vol. 23, p. 321, last index, *for* "$2w-2n$" *read* "$4w-2n$."]

where all the η's represent true aberrations, those on the left relating to the component m and those on the right to the complete lens. This direct additive formula might have been inferred at once from the expression of the aberrations in terms of the coefficients of the standard eikonal with the principal foci as reference points*, and it was in fact in this way that the method of ascertaining the functions appropriate to each component was first constructed.

In a number of applications the aberration of the stop is of no importance, and it may be desired to eliminate it from the equations. This is readily accomplished by giving ξ the special value G in any of the expressions that have been found, the number of independent variables available for the elimination of aberrations being thus reduced from four to three.

Some consideration may now be given to the arrangement of numerical work for the application of the method in a form suitable for the computer who is not a skilled mathematician. As a preliminary a table is constructed showing the heights from the axis at which each lens is met by certain paraxial rays, the first being incident parallel to the axis in the object space, the second parallel to the axis in the space between the first and second lenses, the third parallel to the axis between the second and third lenses, and so on, the last emerging from the last lens parallel to the axis. It is convenient to take the distance of the ray from the axis when it is parallel thereto as unity. The numbers arranged in tabular form are then:

		Number of component lens			
		1	2	3	n
Number of space in which ray is parallel to axis	0	$h_{0,1}$	$h_{0,2}$	$h_{0,3}$	$h_{0,n}$
	1	$h_{1,1}$	$h_{1,2}$	$h_{1,3}$	$h_{1,n}$
	2	$h_{2,1}$	$h_{2,2}$	$h_{2,3}$	$h_{2,n}$
	...	.....			
	n	$h_{n,1}$	$h_{n,2}$	$h_{n,3}$	$h_{n,n}$

where the n heights for each ray are entered horizontally, and $h_{p,q}$ denotes the height at which the qth lens is met by the ray parallel to the axis in the pth medium, that is, between lenses p and $p+1$. By the convention just mentioned $h_{p,p} = h_{p,p+1} = 1$. It will be observed that the complete table may be constructed from the traces of two paraxial rays, and these will usually be those incident and emergent parallel to the axis for the complete instrument, corresponding to the entries in the first and last rows. In terms of the intercepts for these rays

$$h_{p,q} = \frac{h_{0,q}\,(h_{n,p} - h_{n,p+1}) + h_{n,q}\,(h_{0,p+1} - h_{0,p})}{h_{n,p}\,h_{0,p+1} - h_{n,p+1}h_{0,p}} \qquad \ldots\ldots(19).$$

Instead of regarding the table as a set of related quantities placed in rows attention is to be directed to the columns. The ray parallel to the axis in space m and at unit distance from the axis meets component p, or more exactly its unit surfaces, at the height $h_{m,p}$. Thus in medium m the magnification of the image of lens p is $1/h_{m,p}$, and the magnification in the portion of the system following lens m is therefore

$$\frac{h_{m,p}}{h_{n,p}};$$

* See *Trans. Opt. Soc.* 18 (1916–17) 181, or 23 (1921–22) 321, equation (28).

in other words the quantities $h_{0,p}$, $h_{1,p}$, $h_{2,p}$, etc., in column p are proportional to the magnification of the image of lens p in the entire system, after the first lens, after the second lens, and so on. If one of the sets of arbitrary magnifications, say the λ's, is identified with the series of magnifications in the pth column, so that λ_m is proportional to $h_{m,p}$, the coefficient of η_4 for lens p will have a zero coefficient in the general equation, a relation thus being obtained which is independent of this aberration. If the μ's are made equal to the λ's, η_3 for this lens will also disappear. The reason for the construction of the table is now clear. The working quantities in the equations are the sums and differences of the numbers in successive rows, and, for the complete lens, the differences between the first row and the last row after the latter has been multiplied in turn by S and G.

For the purposes of a general investigation it is sometimes preferable to start from an arbitrary table of values for the h's which satisfy (19) rather than from a system of lenses given in power and position. When this is done the elements from which the system is constructed may be derived directly from the table, for the theory of paraxial rays shows that

$$h_{p,r}h_{q,s} - h_{q,r}h_{p,s} = -\frac{\partial^2\kappa_{r,s}}{\partial\kappa_r\partial\kappa_s}\kappa_{p+1,q},$$

thus if the power of the complete lens is κ,

$$\frac{\kappa_p}{\kappa} = \frac{h_{p-1,1}h_{p,n} - h_{p,1}h_{p-1,n}}{h_{0,1}h_{n,n} - h_{n,1}h_{0,n}}$$

and

$$t_p\kappa = h_{0,p}h_{n,p+1} - h_{0,p+1}h_{n,p}$$

As an example consider the incomplete table

Table I

1	·7	·9
1	1	...
...	1	1
·6	·5	1

and suppose it relates to a lens of unit focal length. The above formula at once gives

$$t_1 = 1 \times {\cdot}5 - {\cdot}7 \times {\cdot}6 = {\cdot}08 \quad \text{and} \quad t_2 = {\cdot}7 \times 1 - {\cdot}9 \times {\cdot}5 = {\cdot}25.$$

The powers may be derived from

$$\kappa_1 = \frac{1 - {\cdot}7}{t_1} = 3{\cdot}75,$$

$$\kappa_2 = -\frac{1 \times 1 - {\cdot}6 \times {\cdot}9 - t_1 - t_2}{t_1 t_2} = -6{\cdot}5,$$

$$\kappa_3 = \frac{1 - {\cdot}5}{t_2} = 2,$$

and the missing quantities are,

in the first column, $1 - t_1\kappa_2 = 1{\cdot}52$,

in the last column, $1 - t_2\kappa_2 = 2{\cdot}625$,

agreeing with the values obtainable from formula (19). From the now complete table the working quantities are derived by addition and subtraction. In either case the number of rows is diminished by one, and the successive derived rows may be numbered consecutively 1, 2, 3, ..., row m being associated with component m. The columns may be headed a, b, c, ... More particularly the differences may be distinguished by these letters, and the corresponding capitals used for the sums divided by the corresponding differences.

Table II

Lens	a	b	c
1	0	·3	1·725
2	·52	0	−1·625
3	−·92	−·5	0

Table III

0	·08	·46
−·08	0	·25
−·46	−·25	0

Table IV

2	1·7	3·525
2·52	2	3·625
2·12	1·5	2

Table V

Lens	A	B	C
1	∞	5·667	2·043
2	4·846	∞	−2·231
3	−2·304	−3	∞

Table VI

15·42
5·23
−10·61

Table VII

82·60
−14·78
41·95

Table VIII

A_2	C_2
∞	9·75
7·46	·38
−7·61	∞

Table II shows the differences between successive rows of Table I, and Table III gives the results of dividing each element of a row by the appropriate component power. A satisfactory check on the accuracy of the original table is obtained by seeing that this table resembles a skew-symmetric determinant, the elements of which may be evaluated directly from the first and last rows of the original table. Table IV gives the sums derived from Table I, Table V these quantities divided by the corresponding numbers of Table II. Disregarding those cases in which the value is infinitely great, for the special case of three components three further tables are useful. In Table VI twice the sum of the finite elements on a row of Table V is given. For Table VII the squares and products of the finite elements of Table V are required; for this case these necessarily enter into the equations in the form

sum of squares of finite elements + four times the product

which may be more readily evaluated as

$\frac{3}{2}$ square of sum of finite elements $-\frac{1}{2}$ square of their difference.

For Table VIII only two columns need be constructed. The entries are twice the element of the selected column of Table V plus the remaining finite element of the same row. In this case the first and last columns have been chosen.

The quantities required for the lens as a whole must now be considered, and definite aberrational conditions must be imposed. Let it be required to obtain a lens free from all aberrations in the primary plane for $S = 1$ and $G = 0$. For the construction of the quantities corresponding to those of Table II the last line of Table I has to be multiplied by S and the first line subtracted from it. Corresponding to Table IV the last line is multiplied by G and subtracted from the first line. Just as for Table V, the latter set of quantities is to be divided by the former.

The new tables may be distinguished by the old numbers followed by an accent. Thus

Table II'			Table IV'			Table V'		
a	b	c				A	B	C
$-\cdot4$	$-\cdot2$	$\cdot1$	1	$\cdot7$	$\cdot9$	$-2\cdot5$	$-3\cdot5$	9

The general equation written in full is

$$\Sigma \frac{\lambda_m \mu_m \nu_m \xi_m}{\kappa_m} \left\{ \gamma_m - \beta_m \sigma_m + \left(\frac{1}{2} + \frac{\varpi_m}{3} \right) \tau_m \right\}$$
$$+ \frac{\lambda \mu \nu \xi}{(S-G)^3 \kappa} \{ (1-G)^2 \sigma + 2(1-SG)\tau + 3(1-S)^2 v \} = 0,$$

where $\sigma_m = \Lambda_m + \mathrm{M}_m + \mathrm{N}_m + \Xi_m,$

$$\tau_m = \Lambda_m \mathrm{M}_m + \Lambda_m \mathrm{N}_m + \Lambda_m \Xi_m + \mathrm{M}_m \mathrm{N}_m + \mathrm{M}_m \Xi_m + \mathrm{N}_m \Xi_m.$$

σ and τ are the corresponding quantities without suffix, and

$$v = \Lambda\mathrm{MN} + \Lambda\mathrm{M}\Xi + \Lambda\mathrm{N}\Xi + \mathrm{MN}\Xi.$$

Each λ, μ, ν, or ξ must be identified with a, b, or c, Λ, M, N, or Ξ being of course identified with the corresponding A, B, or C. The a's, etc., with suffixes are given in Table II, those without suffixes in Table II'. The corresponding A's are found in Tables V and V'. Tables VI, VII, and VIII give values of the σ_m's and τ_m's required in the actual equations.

When one of the capital letters represents an infinitely great quantity, the corresponding small letter represents zero, and the product is equal to 2. On the left side of the equation the coefficient only of this capital letter must therefore be retained and the small letter replaced by 2. This procedure covers all cases. For brevity the formula resulting from the substitution

$$\lambda = \mu = a, \quad \nu = b, \quad \xi = c$$

may be denoted by a^2bc. By reference to the numerical tables the following equations for the complete determination of the constituent lenses are readily verified.

Formula b^2c^2

$$(\cdot3)^2 \times 1\cdot725 \times \cdot46 \left\{ \gamma_1 - 15\cdot42\beta_1 + 82\cdot60 \left(\frac{1}{2} + \frac{\varpi_1}{3} \right) \right\}$$
$$+ 4(-1\cdot625) \times \cdot25 \left(\frac{1}{2} + \frac{\varpi_2}{3} \right) + 4(-\cdot5)(-\cdot25) \left(\frac{1}{2} + \frac{\varpi_3}{3} \right)$$
$$+ (-\cdot2)^2 (\cdot1)^2 \{ 2(-3\cdot5+9) + 2 \times \tfrac{3}{2}(-3\cdot5+9)^2 - 2 \times \tfrac{1}{2}(3\cdot5+9)^2 \} = 0.$$

Formula a^2c^2

$$4 \times 1\cdot725 \times \cdot46 \left(\frac{1}{2} + \frac{\varpi_1}{3} \right) + (\cdot52)^2 (-1\cdot625) \times \cdot25 \left\{ \gamma_2 - 5\cdot23\beta_2 - 14\cdot78 \left(\frac{1}{2} + \frac{\varpi_2}{3} \right) \right\}$$
$$+ 4(-\cdot92)(-\cdot46) \left(\frac{1}{2} + \frac{\varpi_3}{3} \right)$$
$$+ (-\cdot4)^2 (\cdot1)^2 \{ 2(-2\cdot5+9) + 2 \times \tfrac{3}{2}(-2\cdot5+9)^2 - 2 \times \tfrac{1}{2}(2\cdot5+9)^2 \} = 0.$$

Formula a^2b^2

$$4 \times \cdot 3 \times \cdot 08\left(\frac{1}{2}+\frac{\varpi_1}{3}\right)+4 \times \cdot 52(-\cdot 08)\left(\frac{1}{2}+\frac{\varpi_2}{3}\right)$$
$$+(-\cdot 92)^2(-\cdot 5)(-\cdot 25)\left\{\gamma_3+10\cdot 61\beta_3+41\cdot 95\left(\frac{1}{2}+\frac{\varpi_3}{3}\right)\right\}$$
$$+(-\cdot 4)^2(-\cdot 2)^2\{2(-2\cdot 5-3\cdot 5)+2\times\tfrac{3}{2}(-2\cdot 5-3\cdot 5)^2-2\times\tfrac{1}{2}(2\cdot 5-3\cdot 5)^2\}=0.$$

Formula abc^2

$$2 \times \cdot 3 \times 1\cdot 725 \times \cdot 46\left\{-\beta_1+9\cdot 75\left(\frac{1}{2}+\frac{\varpi_1}{3}\right)\right\}$$
$$+2 \times \cdot 52(-1\cdot 625) \times \cdot 25\left\{-\beta_2+\cdot 38\left(\frac{1}{2}+\frac{\varpi_2}{3}\right)\right\}+4(-\cdot 5)(-\cdot 46)\left(\frac{1}{2}+\frac{\varpi_3}{3}\right)$$
$$+(-\cdot 4)(-\cdot 2)(\cdot 1)^2\{-2\cdot 5-3\cdot 5+2\times 9+2(-2\cdot 5)(-3\cdot 5)$$
$$+4(-2\cdot 5-3\cdot 5)\times 9+2\times 9^2\}=0.$$

Formula a^2bc

$$4 \times \cdot 3 \times \cdot 46\left(\frac{1}{2}+\frac{\varpi_1}{3}\right)+2(\cdot 52)^2 \times \cdot 25\left\{-\beta_2+7\cdot 46\left(\frac{1}{2}+\frac{\varpi_2}{3}\right)\right\}$$
$$+2(-\cdot 92)^2(-\cdot 25)\left\{-\beta_3-7\cdot 61\left(\frac{1}{2}+\frac{\varpi_3}{3}\right)\right\}$$
$$+(-\cdot 4)^2(-\cdot 2)\times\cdot 1\{2(-2\cdot 5)-3\cdot 5+9+2(-3\cdot 5)9$$
$$+4(-2\cdot 5)(-3\cdot 5+9)+2(-2\cdot 5)^2\}=0.$$

It will be noted that formula ab^2c is equivalent to the elimination of β_2 between abc^2 and a^2bc, so that it is not required. It is for this reason that two columns only of Table VIII were prepared. The number of independent equations is necessarily five because five conditions have been imposed, viz.

$$\eta_0=\eta_1=\eta_2=\eta_3=\eta_4=0 \quad \text{for} \quad S=1,\ G=0.$$

The unknowns, however, number six, the β and γ for each lens. The ϖ may be treated as known, since it varies between such narrow limits. It will be convenient to take β_2 as an independent variable and express the other two β's and the three γ's in terms of it. Each of the ϖ's may be given the value ·63, since this is an average value for lenses of a single glass. If achromatic lenses were likely to be required a value of about ·70 would be appropriate. Upon simplification the equations give

$$\begin{aligned}
\gamma_1 - 15\cdot 42\beta_1 &+ 47\cdot 77 &&= 0,\\
\gamma_2 - 5\cdot 23\beta_2 &- 42\cdot 06 &&= 0,\\
\gamma_3 + 10\cdot 61\beta_3 &+ 30\cdot 88 &&= 0,\\
\beta_1 - \cdot 8874\beta_2 &- 8\cdot 224 &&= 0,\\
\beta_3 - \cdot 3195\beta_2 &+ 18\cdot 324 &&= 0,
\end{aligned}$$

and since the minimum value of γ, denoted by γ_0, satisfies

$$\gamma=\gamma_0+\frac{1+2\varpi}{(1+\varpi)^2}\beta^2,$$

$$(\gamma_0)_1 = \quad 21{\cdot}51 + 1{\cdot}26\beta_2 - {\cdot}67\beta_2^2,$$
$$(\gamma_0)_2 = \quad 42{\cdot}06 + 5{\cdot}23\beta_2 - {\cdot}85\beta_2^2,$$
$$(\gamma_0)_3 = -122{\cdot}07 + 6{\cdot}57\beta_2 - {\cdot}09\beta_2^2.$$

The magnitudes of these values of γ_0 for moderate values of the β's indicate that all the lenses would necessarily be compound, the first if cemented having a positive component of high refractive index with a negative of lower index, the second and third with negative components of high index and positives of lower index. All the components would be decidedly abnormal, as would be expected from a consideration of their powers. In case it were desired to construct such a system suitable glasses could be selected from tables or by the aid of charts described by the author in an earlier paper*. The present example has been included to show in detail the application of the method, and is intentionally of no practical interest in itself.

DISCUSSION.

Comdr Baker (partly communicated): It is very difficult at short notice and with limited time at one's disposal to form a complete and correct view of Mr Smith's paper. The subject-matter is so clearly bound up with his earlier papers on similar subjects that I find it hard, not having his nomenclature at my fingers' ends, to follow the paper in detail and I am very diffident of making any criticism except on general lines.

As far as I am able to judge, this paper makes a very big advance in the methods of optical design and should, when the details of Mr Smith's methods are understood by the computing optician, be of great service to him in his work. It will not furnish much assistance to the designer who computes entirely by trigonometry and who trusts to his expert knowledge and to a blind groping for solutions by trial and error methods. But for the man who bases his optical design on the assumption that the initial step is to clear the first order aberrations there seems to be in this paper a reasoned *modus operandi*. It is time that we had such a scheme available. So far text-books show us how to evaluate the first order aberrations of a single lens and to express them in terms of a parameter, such for instance as the lens shape. They show us how to sum up aberrations through a lens system and in theory we could write down five equations each containing as unknown quantities, quadratic in form, the lens shapes of the components of the system. And there in general the matter is left. No attempt is made to show how, when each of these first order aberrations is put equal to zero, the solutions for the lens form are to be obtained. Mr Smith now produces such a scheme and I think that when we all understand more clearly exactly what he is doing we shall find that he has given in this paper a very valuable piece of work.

I am sure, however, that we all of us need more numerical illustrations to be able to follow his methods. I suggested at the meeting and I still think it is a desirable thing that he should give to the Society a complete scheme of calculation for some optical system with which optical computers are already familiar. I should like to

* *Trans. Opt. Soc.* 19 (1917–18) 169.

see the Kellner eyepiece worked out in such a manner taking, say, for his principal conditions an equivalent focal length of one inch, the front focus to be one-tenth of an inch in front of the field lens and back focus four-tenths of an inch behind the eye lens, the exit pupil being one-tenth of an inch further. Such a system corresponds fairly closely with Kellner eyepieces that are now made and with which manufacturers are thoroughly familiar. If Mr Smith will work out such a case and can show that his methods lead to curves such as manufacturers are in the habit of using, the example will do far more to bring home to them the advantages of employing the scheme than a general exposition such as is contained in his paper or than an example taken from a three lens system whose performance is unfamiliar to them.

Although in my opinion Mr Smith's scheme is of great practical value when the preliminary work of design is based upon removal of first order aberrations, yet it does not enable a design to be carried out entirely by consideration of aberrations alone. When the first order is corrected there remain still the aberrations of higher order and these can still only be compensated for by first determining their values and secondly so modifying those of the first order that they will balance the residuals. If the variables that Mr Smith employs are such that when the first order terms go those of higher orders also disappear or must inevitably be very small, the scheme that he propounds is more valuable still, but I am not clear on this point and should like further information. Unless it is possible by a proper choice of variables to ensure this point I do not think that we shall have arrived at a position where optical instruments can be designed by computation of aberrations alone without reference to ray tracing in some form or other.

Mr Smith: As Commander Baker has pointed out, there is nothing in this paper which removes the necessity for further calculations by one or other of the recognised methods to complete the design of an optical instrument. The special problem discussed here happens to be of interest whatever method is adopted for the completion, and in particular, while the solution may enable much time to be saved by a computer working on traditional lines, it is essential to any scheme which aims at the abolition of ray tracing as a method of design. The subject has been investigated as one of a series of problems involved in a theory of design for well corrected lenses which does not call for any ray tracing. These further problems are not of so simple a character that they can be appended to a paper like the present one. So while it is hoped that the work will ultimately assume the greater importance to which Commander Baker refers, at the present time the lower valuation must be accepted.

As regards further illustrations, that suggested by Commander Baker is not very attractive for the reason that the Kellner eyepiece is not a particularly well corrected instrument. It is hoped, however, to submit on a future occasion the results of some calculations by this method which may serve the purpose Commander Baker has in mind.

EXHIBIT OF A NEW RESEARCH MICROSCOPE

By J. H. BARTON

Exhibited, 12*th April*, 1923.

The main object of this microscope is to give the user the advantage of being able to employ first class condensers and high power dark ground illuminators with the same ease and facility as with the much less efficient Abbe condenser. The aplanatic cone of the Abbe is much smaller than that of a first class condenser and the result is that when using the Abbe with high powers much of their efficiency is lost for the reason that a very appreciable portion of their actual N.A. is non-effective.

In addition, however, to the facility it affords in the use of first class condensers there are other features which allow of pleasant and rapid working with a wide range of adaptability.

The base is original and combines the advantages of both the horse shoe and the tripod without entailing any new disadvantage. It is supported on *four* toes all of which readily adapt themselves to any uneven surface. The back pair of toes are almost as wide apart as the front pair and the stability thus gained is markedly greater than in any other form of base yet devised.

Any form of body tube can be used, either monocular or binocular, and may be interchanged in a few seconds.

The slides are very broad and not sprung, but, should wear eventually occur, it can readily be corrected by an improved method of construction.

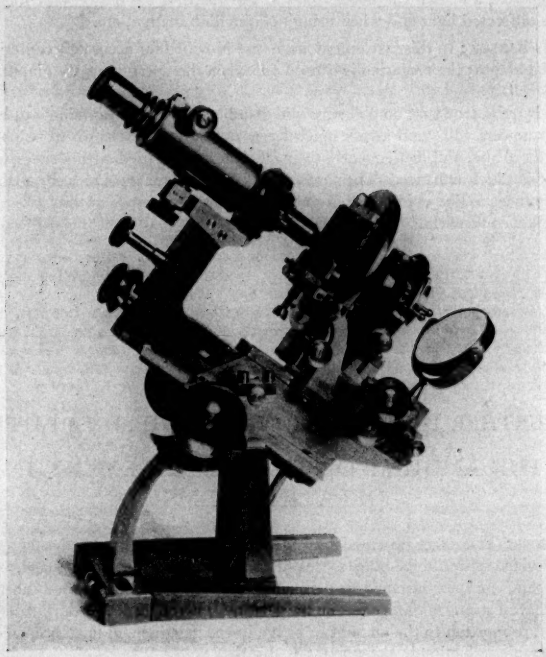
The stage fitted on the instrument shown is circular and rotating, but any kind of stage can be fitted according to the class of work for which it is intended. Whatever form of mechanical stage is fitted, the milled heads controlling it are below the surface of the top plate, and in order therefore to obtain a plain stage nothing has to be removed except the object holder.

The substage differs considerably from the usual pattern, and on it are mounted the means whereby the various methods of illumination are available immediately without having to lose time by the changing of apparatus.

The fine adjustment is on the long lever principle, and consists of a dovetail slide fitting which is independent of the body. There are two speeds, one of which is very slow and the second one, got by displacing the fulcrum, gives one-fiftieth of an inch motion for each turn of the milled head.

The means of maintaining the alignment of the optical portions have been much simplified, and the iris diaphragm of the substage is correctly centered to the optical axis of the body.

All the mechanical parts have been designed with a view to the cheapening of production by the use of machine tools, and the fitting of the racks is effected in a novel manner, equally as effective but much less costly than the usual methods.



DISCUSSION.

Mr J. GUILD: It is a pleasure to see an instrument designed almost entirely from first principles in complete disregard of conventional design. There is no instrument in the design of which convention plays so large a part as the modern microscope.

As regards the stress laid by Mr Barton on accurate centering of the substage, has he ever come across an iris diaphragm which is concentric about the same point for all openings?

Then as regards the importance of using very high class condensers, my experience (which is possibly not sufficiently extensive to generalise from) is that few, if any, of the best high power objectives will stand anything like a full cone of

illumination, and that the outer zones of the aperture are only useful for letting in the diffracted light and when using oblique illumination, etc.

Mr BARTON: In this instrument means are provided for accurately centering the iris diaphragm; the centering is carried out when the aperture of the diaphragm is very small.

It is quite true that no objective will stand a full cone of illumination, but the higher powers will stand a very much larger aplanatic cone than is to be obtained from the Abbe, and consequently their efficiency when used with a superior condenser is much enhanced. The preference for the Abbe appears to be mainly its facility in use. The object of this microscope is to give equal facility when using high class condensers.

FURTHER DISCUSSION ON Mr DAVIDSON'S PAPER

THE AMOUNT OF THE DISPLACEMENT IN GELATINE FILMS

Mr STORR: The effect observed by Mr Guild, as compared with that shown by Mr Davidson, would be due to the difference in the developer used. Some developers, such, for instance, as pyrogallic acid, on oxidation produce substances which have a strong tanning effect on the gelatine. This effect occurs during development in proportion to the amount of image in the gelatine and is accompanied by considerable reduction in the amount of absorbed water. The relief effect obtained in a wet negative is, therefore, very pronounced. I was not aware, however, until I examined carefully some negatives produced by such a developer, that the relief effect persists in the same direction, though only slightly, after the negative is dried.

[Correction: On p. 46 of the present volume, 5th line from the bottom, *for* "bromide" *read* "silver."]

REVIEW

Generalised Linear Perspective, treated with special reference to Photographic Land Surveying and Military Reconnaissance. By J. W. GORDON. pp. xvi + 184; 51 Figs. (London: Constable and Co. Ltd., 1922.) Price 21*s.* net.

The author aims to specify a rational procedure for the preparation of plans from oblique photographs, taken say from aeroplanes, and other related problems in which the plane on which the projection is made is neither horizontal nor vertical. This was worked out in ignorance of Brook Taylor's book published more than two hundred years ago. For the mathematician the book contains no novel matter and is wanting in brevity; the accepted nomenclature is discarded and a new one coined; unnecessary definitions are introduced, and the wording is in many places obscure. To the artist that may be a merit which to the mathematician is a defect, and if so this book may meet a real need. We fear however that such readers will find the use of circular functions and their relations a stumbling block, which is the more unfortunate as the treatment could have been simplified and abbreviated by their omission. In fact the theory can easily be based solely on the properties of similar triangles, properties likely to commend themselves as almost obvious to those who have had no instruction in geometry. The whole algebraic theory likewise is contained in equations of the form

$$\frac{\eta}{y} = \frac{f}{x-a} = \frac{\xi - a}{f'}$$

in which there are decided advantages in retaining the distinction between f and f'. This equation shows what is otherwise obvious, that the problem is identical with that of the simplest case of optical collinear imagery. In this connection the author's statement that this "formidable problem" was "diligently studied by the military staffs of all the belligerent armies during the years of war, and apparently without producing any appreciable impression on the problems themselves, which the latest publications treat as unsolved" will be read with amusement.

Readers interested in the application of the theory to the construction of plans from a series of photographs will find here a well executed illustration. A proposal made by the author for the recording of the vertical in photographs seems suited only for Laboratory experiments.

T. S.

ABSTRACTS OF PATENT SPECIFICATIONS

186240. Orr, E. A. SIGNALS.

The speed-control gear of a motor vehicle is inter-connected with a shutter, which is operated to cut down the illumination of the tail lamp when speed is reduced.

186311*. Kourkene, H. SURVEYING-INSTRUMENTS.

A movable cross-wire is adjusted by means of a disc so as to intercept between itself and a fixed cross-wire the image of a known length of a stadia rod, the position of the disc giving a reading of the distance. The disc also automatically focuses the telescope.

* Specification published before acceptance.

186335*. Précision Moderne. RANGEFINDERS.

The setting of three sliding and rotating pointers in accordance with the magnitude and direction of the motion of the firing vessel, of the target, and of the wind, relatively to the line of sight, operates through a series of inter-related mechanisms to show upon a pair of dials the corrections to the gun sights which have to be applied in elevation and deflection respectively.

186550. Elorza, J. de. ILLUMINATED SIGNS.

Hollow letters or objects are made of thick glass, and illuminated from within. A current of steam fed into the interior issues through slots in the glass, and is illuminated by red light, so as to simulate burning.

186575*. Aktieselkapet Autopil. VEHICLE LAMPS; SIGNS.

To soften the light from a vehicle lamp, a semi-transparent sheet is enclosed between two glass plates to form the front of the lamp. For advertising purposes, a sheet bearing the required matter may be enclosed between the front glass of a lantern and a cover-glass.

186615. Bouchaud-Praceiq, E. PREVENTING CONDENSATION.

Glass surfaces are protected against the condensation of moisture thereon by wiping over with soft paper, in which has been incorporated an alkaline soap, together with oil-absorbents such as alkaline-earth carbonates or borates, or kieselguhr.

186648. Yorke, F. A. RANGEFINDERS.

The instrument includes a mechanical triangle of which the observing telescope, rotating about the angle corresponding to the target, forms one side, the second side consisting of a bar which is set parallel to the line observer-gun, in accordance with the indication of an electric controlling-device. The third side of the triangle gives the range and bearing of the target.

187271. Vickers, Ltd. and another. PERISCOPES.

An additional prism is placed above the ordinary observing prism, from which it is separated by a small space. By filling this space when desired with a liquid of the same refractive index as the glass, objects immediately overhead may be observed.

187613. General Electric Co., Ltd. ELECTRIC DISCHARGE LAMPS.

The globe encloses a transparent body which, when subjected to electron bombardment, emits radiation in the desired region of the spectrum.

188132. Schofield, F. H. PHOTOMETERS; PYROMETERS.

In an instrument in which the intensity of a source is balanced by varying the current through a lamp, the latter is placed in one arm of a bridge, and a galvanometer in the cross connection. The lamp, bridge resistances, and galvanometer series resistance are all put up in a single replaceable unit, and the whole range of the galvanometer, beginning at any suitable lamp current, can be made use of.

188313*. Zeiss, C. MICROSCOPES.

A negative lens system is fitted within the tube of a microscope, in advance of a prism having a half-silvered surface by which part of the light is reflected into a camera, while the remainder is transmitted to an eyepiece mounted on an extension of the tube, so that the object can be observed while the photograph is being taken. R. J. T.

* Specification published before acceptance.